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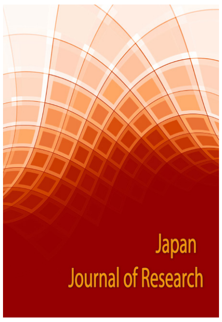

Entropy and Fisher Information in Process Viability: The IEPI Framework

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Abstract

Entropy quantifies informational dispersion in stochastic systems, whereas Fisher information measures local sensitivity to parametric perturbations. This paper introduces the Information Entropy Performance Indicator (IEPI), a discrete information-theoretic framework that jointly characterises uncertainty and responsiveness to assess the viability of stochastic processes. A functional entropy range [H_min^*,H_max^*] defines a bounded regime between deterministic rigidity and stochastic instability. Analytical results establish a formal coupling between Shannon entropy and Fisher information, yielding a responsiveness-induced entropy floor and enabling precise viability criteria. IEPI thus provides a mathematically rigorous basis for steering adaptive processes within defined informational limits.

Introduction

Information-theoretic measures play a central role in the quantitative analysis of uncertainty, variability, and system structure [1–4]. Shannon entropy provides a canonical scalar descriptor of informational dispersion in stochastic systems [5,6], whereas Fisher information quantifies local sensitivity to parametric perturbations [7–9]. The interaction between these quantities underpins fundamental results in statistics, information geometry, and control theory.

This paper introduces the *Information Entropy Performance Indicator* (IEPI), a formal framework for assessing the viability of stochastic processes subject to bounded informational uncertainty. The approach integrates global dispersion and local responsiveness into a unified analytic construct, reflecting the principle that systems must retain sufficient uncertainty to remain adaptive while preserving sufficient sensitivity to remain controllable. In this sense, IEPI delineates an *informationally stable zone* within which sustainable operation can occur.

Previous research has examined entropy-based complexity indices or information-flow metrics independently (e.g. [1–4]), typically treating entropy and Fisher information as separate descriptors. The present contribution differs by establishing a mathematically coupled relationship between these quantities and deriving sharp viability bounds from their joint behaviour. This unified information-theoretic formalism supports both descriptive analysis and regulatory decision-making, enabling systematic evaluation and redesign of uncertainty-generating structures.

The primary objective of this work is to provide a rigorous foundation for IEPI as a performance indicator in discrete stochastic environments and to motivate further mathematical development and domain-specific application. The coupling between Shannon entropy, Fisher information, and Kullback–Leibler divergence [9] defines a coherent sensitivity–dispersion relation through which viability under uncertainty can be characterised, monitored, and improved within a single information-theoretic framework.

Section 2 presents the mathematical preliminaries: entropy, effective support, and the functional entropy range. Section 3 develops the core results connecting entropy, responsiveness, and information geometry, leading to feasibility conditions and compositional stability. Section 4 demonstrates IEPI-based assessment in representative control-flow structures and discusses practical implications. Section 5 concludes with a synthesis of contributions and prospective research directions.

Preliminaries

This section introduces the core definitions and mathematical relationships used throughout the IEPI formulation. All random variables are assumed to be discrete with finite support $X = \{x_1, \ldots, x_n\}$ of size n. The probability mass function is denoted $p_i = \Pr(X = x_i)$, with $\sum_{i=1} p_i = 1$. All logarithms are taken to the natural base unless otherwise indicated; entropy values may be expressed in bits by dividing by ln 2.

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Notation

Throughout this paper, the (n-1)–dimensional probability simplex is defined as

$$\Delta_{n-1} = \left\{ p \in \mathbb{R}^n_{\geq 0} : \sum_{i=1}^{n} p_i = 1 \right\}.$$

The Shannon entropy of a distribution p is denoted *H(p)*, and the associated effective support size (perplexity) is

$$S_{\text{eff}}(p) = \exp\big(H(p)\big).$$

The constants $H^*_{\min}$ and $H^*_{\max}$ represent prescribed lower and upper entropy bounds defining the viable operational range for a process. For a parametric family *p(;θ), I(θ)* denotes the Fisher Information Matrix, and its trace

$$R(\theta) = \operatorname{tr}\big(I(\theta)\big)$$

is referred to as scalar responsiveness, capturing the aggregate local sensitivity to perturbations in θ.

Definition 1 (Shannon entropy). Let X be a discrete random variable supported on X={x_1,…,x_n} with probability mass function p_i=Pr(X=x_i). The Shannon entropy of X is defined as

$$H(X) = -\sum_{i=1}^{n} p_i \log p_i.$$

It satisfies the bounds

$$0 \leq H(X) \leq \log n,$$

with equality on the left if and only if *X* is almost surely constant, and equality on the right if and only if $p_i = \frac{1}{n}$ for all i.

Remark 1 (Analytical properties of Shannon entropy). The Shannon entropy H is non-negative on Δ_{n-1} and satisfies *H(p)=0* if and only if *p* is degenerate at a single point. It is additive under independence: if *X* and *Y* are independent random variables, then *H(X,Y)=H(X)+H(Y)*.

The function *H* is strictly concave on the relative interior of Δ_{n-1} . For any p, q$\in\Delta_{n-1}$ and λ∈(0,1),

$$H(\lambda p + (1-\lambda)q) \geq \lambda H(p) + (1-\lambda)H(q),$$

with equality if and only if *p = q*.

At any interior point with $p_i > 0$, the gradient and Hessian satisfy

$$\nabla_i H(p) = -\log p_i - 1, \qquad \nabla^2 H(p) = -\operatorname{diag}(1/p_i).$$

The Hessian is negative definite when restricted to the tangent space $\{v \in \mathbb{R}^n : \sum_i v_i = 0\}$\$,

which confirms strict concavity under the simplex constraint.

Proposition 1 (Concavity and bounds). Let $p \in \Delta_{n-1}$ with $n \geq 2$. Then $0 \leq H(p) \leq \log n$, with $H(p) = 0$ if and only if *p* is a vertex of Δ_{n-1}, and $H(p) = \log n$ if and only if $p_i = \frac{1}{n}$ for all *i*. Moreover, *H* is strictly concave on every face where $p_i > 0$.

Proof. Let p$\in\Delta_{n-1}$.

Lower bound and equality. For x ∈ [0,1], the inequality $x\log x \geq 0$ holds, with the convention *0log : = 0* by continuity. Therefore,

$$H(p) = -\sum_{i=1}^{n} p_i \log p_i \geq 0.$$

If *H(p) = 0*, each term $p_i \log p_i$ must vanish, which occurs only when a single coordinate equals 1 and all others are zero. Conversely, any such degenerate vertex of Δ_{n-1} yields *H(p) = 0*.

Upper bound and equality. To determine the maximum of H under the normalization constraint $\sum_i p_i = 1$, consider the Lagrangian

$$\mathcal{L}(p,\lambda) = -\sum_{i=1}^{n} p_i \log p_i + \lambda\Big(\sum_{i=1}^{n} p_i - 1\Big).$$

The first-order conditions satisfy

$$-(\log p_i + 1) + \lambda = 0,$$

so $p_i = e^{\lambda - 1}$ for all i. Summing over coordinates yields $p_i = \frac{1}{n}$, implying

$$H(p) \leq \log n,$$

with equality if and only if p is uniform. Boundary points cannot attain this maximum.

Strict concavity. For $p_i > 0$, the Hessian satisfies $\nabla^2 H(p) = -\operatorname{diag}(1/p_i)$. Thus, for any nonzero *v* in the tangent space $\{v : \sum_i v_i = 0\}$,

$$v^\top \nabla^2 H(p) v = -\sum_{i=1}^{n} \frac{v_i^2}{p_i} < 0,$$

confirming strict concavity on the relative interior of Δ_{n-1}.

Definition 2 (Effective support and perplexity).For interpretability, one may introduce a thresholded support

$$S_\varepsilon(X) = \{x_i \in X : p(x_i) > \varepsilon\}, \qquad \varepsilon > 0,$$

representing the set of outcomes with non-negligible probability. The effective cardinality (perplexity) of p is defined as

$$S_{\text{eff}}(p) = \exp\big(H(p)\big),$$

which equals *n* when *p* is uniform on n outcomes and more generally represents the number of equally likely states that would produce the same entropy.

Remark 2 (Interpretation): The magnitude of S_{eff} *(p)* quantifies how widely probability mass is dispersed. A small value $S_{\text{eff}}(p) \ll |X$ indicates concentration on a limited set of outcomes, while $S_{\text{eff}}(p) \approx |X||$reflects a distribution close to uniformity. Perplexity therefore provides an interpretable scale for variability and complements entropy by mapping it to an effective number of likely states.

Definition 3 (Functional entropy range): Let and H_max^* be prescribed entropy thresholds satisfying

$$0 < H^*_{\min} < H^*_{\max} \leq \log n.$$

The viable entropy band is defined as

$$H_{\text{viable}} = \left\{ p \in \Delta_{n-1} : H^*_{\min} \leq H(p) \leq H^*_{\max} \right\},$$

representing the admissible range between deterministic collapse and maximal stochastic dispersion.

Remark 3 (Viability band): The interval [0,logn] describes the full admissible entropy range. The functional sub-band $[H^*_{\min}, H^*_{\max}]$ identifies configurations that preserve a sustainable balance between predictability and flexibility, excluding both deterministic collapse and uncontrolled stochastic dispersion.

Definition 4 (Fisher information and responsiveness): Let $\{p(x_i;\theta)\}_{i=1}^n$ be a discrete parametric family with parameter $\theta \in \Theta \subset \mathbb{R}^k$. Following Fisher [6], the *Fisher Information Matrix* is defined as

$$I(\theta) = \sum_{i=1}^{n} p(x_i;\theta)\big[\nabla_\theta \log p(x_i;\theta)\big]\big[\nabla_\theta \log p(x_i;\theta)\big]^\top.$$

Its trace,

$$R(\theta) = \mathrm{tr}\big(I(\theta)\big),$$

is termed *responsiveness*, as it quantifies the aggregate local sensitivity of the distribution to infinitesimal parameter perturbations. Since *tr(I(θ))* depends on the choice of parametrization, a coordinate-invariant scalarization such as

$$R_G(\theta) = \mathrm{tr}\big(G^{-1/2} I(\theta) G^{-1/2}\big)$$

may be adopted for a positive definite metric *G*.

In settings where no explicit parameter *θ* is modelled, an operational surrogate of responsiveness is given by

$$R(p) = 1 - \sum_{i=1}^{n} p_i^2,$$

which interprets responsiveness directly through the induced categorical probabilities.

Remark 4 (Responsiveness and adaptivity): Large values of *R(θ)* indicate strong sensitivity to perturbations in the underlying parameters, corresponding to enhanced adaptive capability. Conversely, *R(θ)≈0* signals rigidity, since the distribution changes little under infinitesimal control variation. The Fisher information therefore provides a quantitative measure of the capacity of a process to react to and exploit available control directions.

Lemma 1 (Entropy–responsiveness link for categorical splits): Let $p = (p_1, \ldots, p_n) \in \Delta_{n-1}$ be parameterised via logits $\theta \in \mathbb{R}^n$ by $p_i = \exp(\theta_i)/\sum_j \exp(\theta_j)$. *Since the associated score vector is* $s(\theta; x_i) = e_i - p$, *the Fisher Information Matrix is*

$$I(\theta) = \mathbb{E}[ss^\top] = \mathrm{diag}(p) - pp^\top, \qquad R(p) = \mathrm{tr}(I(\theta)) = 1 - \sum_{i=1}^{n} p_i^2.$$

Let $H_2(p) = -\log\left(\sum_{i=1}^{n} p_i^2\right)$ *denote the Rényi-2 (collision) entropy. Since* $H(p) \geq H_2(p)$ *for all* $p \in \Delta_{n-1}$, *we obtain* $H(p) \geq H_2(p) = -\log\big(1 - R(p)\big)$. *Because* $\sum_i p_i^2 \in [1/n, 1]$, *we have* $R(p) \in [0, 1 - 1/n]$ *and the right-hand side of [eq:bound] is strictly increasing in R(p) over this interval. For any fixed value of R(p), equality holds if and only if H(p)=*H_2 *(p), which occurs precisely when p is uniform on its support.*

Remark 5 (Conceptual link): Entropy characterises the global dispersion of probability mass, whereas Fisher information quantifies the local sensitivity of the distribution to perturbations of its governing parameters. Inequality [eq:bound] therefore establishes that a minimum level of responsiveness induces a minimum level of uncertainty, linking adaptability and variability through a single analytical constraint. Bounds on R and H together delineate the viable operating regime that IEPI seeks to maintain.

These constructs complete the informational foundation on which IEPI is built. Entropy and responsiveness describe complementary geometric aspects of variation, and their quantified interaction provides the structural basis for viability. The next section develops the analytical results that formalise this interplay, including the feasibility region, responsiveness–implied entropy floors, and closure under compositional design operations.

Main Results

All logarithms are natural unless otherwise stated; entropy values expressed in bits are obtained by dividing by ln2. For notational convenience, define the normalized quantities.

$$\eta := \frac{H(X)}{\log n} \in [0,1], \qquad \rho := \frac{R(\theta)}{R_{\max}} \in [0,1], \qquad R_{\max} = 1 - \frac{1}{n}.$$

Here, *η* represents the relative entropy and ρ the normalized responsiveness. A system is said to be viable when

$$\eta \in [\eta^*_{\min}, \eta^*_{\max}], \qquad \rho \geq \rho^*_{\min},$$

where $\eta^*_{\min}, \eta^*_{\max}$, and $\rho^*_{\min}$ denote functional thresholds determined by the operational domain.

When a parametric model *p(·;θ)* is considered, *θ* denotes a fixed coordinate system on the parameter space *Θ*, and all Fisher information quantities are evaluated with respect to this parametrization.

Entropy-bounded viability

A stochastic process is said to be entropy-viable if

$$H^*_{\min} \leq H(X) \leq H^*_{\max}.$$

The lower bound corresponds to collapse into near-deterministic rigidity, while the upper bound corresponds to stochastic divergence, where predictive control becomes ineffective.

Fisher-bounded responsiveness

Let $R(\theta) = \mathrm{tr}(I(\theta))$ denote the scalar measure of responsiveness. A process is said to be *responsiveness-viable* if

$$R(\theta) \geq \rho_{\min} > 0.$$

Low values of *R* indicate insufficient sensitivity to perturbations, signifying a regime in which the system cannot be effectively influenced or controlled.

Remark 6 (Coordinate choice and normalization): Since *tr(I(θ))* depends on the choice of parametrization, a coordinate-invariant scalarization can be adopted:

$$R_G(\theta) = \mathrm{tr}\big(G^{-1/2} I(\theta) G^{-1/2}\big),$$

where *G* is a positive definite reference metric on the parameter space. In the absence of a metric transformation, *R(θ)* is evaluated in the fixed coordinates associated with *θ*.

Entropy–responsiveness coupling

Theorem 1 (Entropy–responsiveness coupling)

Let p ∈ Δ_{n-1} denote the branch probabilities of a categorical split. Then $R(p) = 1 - \| p \|_2^2$, $H(p) \geq -\log\big(1 - R(p)\big)$. Equality holds if and only if $H(p) = H_2(p)$ equivalently when p is either concentrated on a single outcome or uniform on its support.

Proof: By Lemma 1, we have

Since *H(p)* ≥ H_2 *(p)* for all *p* ∈ Δ_{n-1}, the stated entropy–responsiveness bound follows immediately. Equality occurs precisely when *p* is uniform on its support or concentrated on a single outcome, which completes the proof. □

Corollary 1 (Responsiveness floor implies entropy floor)

If *R(p)* ≥ ρ_{min} for some $\rho_{min} > 0$, then the entropy of the process satisfies *H(p)* ≥ - log(*1 -* ρ_{min}).

Proof: Directly substituting *R(p)* ≥ ρ_{min} into the inequality of Theorem 1 yields

$$H(p) \geq -\log\big(1 - R(p)\big) \geq -\log\big(1 - \rho_{\min}\big),$$

which establishes the stated entropy floor. □

Controlled responsiveness and spectral bounds

In many operational settings, only a subset of variability directions is directly influenced by control actions. This motivates a weighted notion of responsiveness. Let $\Sigma_{ctrl} \succcurlyeq 0$ denote a positive semidefinite matrix encoding the controllable directions in the parameter space. The *controlled responsiveness* is defined as

$$R_\Sigma(\theta) = \mathrm{tr}\big(I(\theta)\Sigma_{\mathrm{ctrl}}\big).$$

Using standard spectral inequalities based on the Courant–Fischer theorem [11], one obtains the bounds

$$\lambda_{\min}\big(\Sigma_{\mathrm{ctrl}}\big)R(\theta) \le R_\Sigma(\theta) \le \lambda_{\max}\big(\Sigma_{\mathrm{ctrl}}\big)R(\theta),$$

where λ_{min} and λ_{max} denote respectively the smallest and largest eigenvalues of Σ_{ctrl}. Thus, controlled responsiveness scales proportionally to the intrinsic responsiveness $R(\theta)$, up to known spectral constants determined by control structure.

Remark 7 (Partial controllability and spectral interpretation)

The matrix Σ_{ctrl} acts as a spectral filter that restricts responsiveness to the subspace of actively regulated directions. Equation [eq:spectral_bounds] demonstrates that the degree of controllable adaptivity is scaled between the smallest and largest eigenvalues of Σ_{ctrl}. When $\lambda_{\min}(\Sigma_{ctrl}) = 0$, certain variability modes are inherently uncontrollable, representing informational inertia. Conversely, large $\lambda_{max}(\Sigma_{ctrl})$ amplifies responsiveness in dominant control directions, which may enhance adaptivity but also increases the risk of instability. Within the IEPI framework, R_Σ is therefore interpreted as a measure of effective control bandwidth, quantifying how adaptivity is distributed across controllable dimensions while ensuring sustainable stability.

Local divergence geometry and curvature bounds

Fisher information quantifies the expected local curvature of statistical distance, describing how rapidly a distribution changes under infinitesimal perturbations of its control parameters. Its geometric interpretation arises naturally from the second-order expansion of the Kullback–Leibler divergence when comparing nearby parameter values [10, 6].

Proposition 2 (Average KL divergence)

Let u be uniformly distributed on the unit sphere in $\mathbb{R}^k$. For sufficiently small

$$\delta > 0, [\mathbb{E}_u\Big[D_{\mathrm{KL}}\big(p(\cdot;\theta)\big\|p(\cdot;\theta+\delta u)\big)\Big] = \frac{\delta^2}{2k}\mathrm{tr}\,I(\theta) + o(\delta^2).$$

Proof. Expanding the Kullback–Leibler divergence to second order in δ around θ gives

$$D_{\mathrm{KL}}\big(p(\cdot;\theta)\|p(\cdot;\theta+\delta u)\big) = \tfrac{1}{2}\delta^2 u^\top I(\theta)u + o(\delta^2).$$

Taking the expectation with respect to u and using $\mathbb{E}[uu^\top] = \frac{1}{k}I_k$ yields the stated result.

Proposition 3 (Average χ^2 sensitivity)

Let u be uniformly distributed on the unit sphere in $\mathbb{R}^k$. For sufficiently small $\delta > 0$, $\mathbb{E}_u\Big[\chi^2\big(p(\cdot;\theta+\delta u), p(\cdot;\theta)\big)\Big] = \frac{\delta^2}{k}\mathrm{tr}\,I(\theta) + o(\delta^2)$.

Sketch of proof. A second-order expansion of the Pearson χ^2 divergence gives $\chi^2(p(\theta+\delta u), p(\theta)) = \delta^2 u^\top I(\theta)u + o(\delta^2)$. Averaging over u on the unit sphere and using $\mathbb{E}[uu^\top] = \frac{1}{k}I_k$ yields the result (cf. [6, 12]).

Proposition 4 (Directional curvature floors)

Let $S \subset \mathbb{R}^k$ be a controllable subspace of dimension $d \ge 1$ with orthogonal projector Π_S. Assume $p(\cdot;\theta)$ is twice continuously differentiable in θ with finite Fisher information $I(\theta)$. Let $\Sigma_{\mathrm{ctrl}} \succeq 0$ satisfy $\mathrm{range}(\Sigma_{\mathrm{ctrl}}) \subseteq S$, $\Sigma_{\mathrm{ctrl}} \succeq \sigma_{\min}\Pi_S$ for some $\sigma_{\min} > 0$. Then, for any unit vector $u \in S$ and sufficiently small $\delta > 0$,

$$D_{\mathrm{KL}}\big(p(\cdot;\theta)\|p(\cdot;\theta+\delta u)\big) = \frac{\delta^2}{2}u^\top I(\theta)u + o(\delta^2) \ge \frac{\delta^2}{2}\lambda_{\min}\big(I(\theta)|_S\big) + o(\delta^2).$$

If, moreover, the Fisher information is uniform on S in the sense that $I(\theta)|_S \succeq \frac{\alpha}{d}\mathrm{tr}\big(I(\theta)\Pi_S\big)\Pi_S$ for some $\alpha \in (0,1]$, then, for any unit $u \in S$,

$$D_{\mathrm{KL}}\big(p(\cdot;\theta)\|p(\cdot;\theta+\delta u)\big) \ge \frac{\alpha\delta^2}{2d}\mathrm{tr}\big(I(\theta)\Pi_S\big) + o(\delta^2).$$

Since $\Sigma_{\mathrm{ctrl}} \preceq \lambda_{\max}(\Sigma_{\mathrm{ctrl}})\Pi$, it follows that

$$\mathrm{tr}\big(I(\theta)\Pi_S\big) \ge \frac{1}{\lambda_{\max}(\Sigma_{\mathrm{ctrl}})}\mathrm{tr}\big(I(\theta)\Sigma_{\mathrm{ctrl}}\big) = \frac{R_\Sigma(\theta)}{\lambda_{\max}(\Sigma_{\mathrm{ctrl}})}.$$

Combining (8)-(9) yields the operational lower bound

$$D_{\mathrm{KL}}\big(p(\cdot;\theta)\|p(\cdot;\theta+\delta u)\big) \ge \frac{\alpha\delta^2}{2d\lambda_{\max}(\Sigma_{\mathrm{ctrl}})}R_\Sigma(\theta) + o(\delta^2).$$

Proof. Assume $p(\cdot;\theta)$ is twice continuously differentiable in θ with finite Fisher information $I(\theta)$, equivalently $\mathbb{E}\big[\|\nabla_\theta \log p(X;\theta)\|^2\big] < \infty$. Along the parametric curve a second-order expansion of the Kullback–Leibler divergence yields, as $\delta \to 0$,

$$D_{\mathrm{KL}}\big(p(\theta)\|p(\theta+\delta u)\big) = \frac{\delta^2}{2}u^\top I(\theta)u + o(\delta^2) \quad \text{(cf. [12])}.$$

For any $u \in S$ with $\|u\| = 1$, the Rayleigh–Ritz principle applied to the restriction $I(\theta)|_S$ implies

$$u^\top I(\theta)u \ge \lambda_{\min}\big(I(\theta)|_S\big),$$

proving (6). If the uniformity condition (7) holds, then for all such u,

$$u^\top I(\theta)u \ge \frac{\alpha}{d}\mathrm{tr}\big(I(\theta)\Pi_S\big),$$

and substitution into the expansion gives (8).

Since $\mathrm{range}(\Sigma_{ctrl}) \subseteq S$, the maximal eigenvalue $\lambda_{\max}(\Sigma_{\mathrm{ctrl}})$ is realized on S, and $\Sigma_{\mathrm{ctrl}} \preceq \lambda_{\max}(\Sigma_{\mathrm{ctrl}})\Pi_S$. By the Löwner order and positive semidefiniteness of $I(\theta)$,

$$\mathrm{tr}\big(I(\theta)\Pi_S\big) \ge \frac{1}{\lambda_{\max}(\Sigma_{\mathrm{ctrl}})}\mathrm{tr}\big(I(\theta)\Sigma_{\mathrm{ctrl}}\big),$$

which establishes (9). Combining this inequality with (8) yields the weighted curvature floor (10) . The argument uses the standard trace monotonicity of positive semidefinite matrices and the Courant–Fischer characterization of eigenvalues (cf. [11,12]).

Isotropic full-subspace case

If $S = {}^k, \Sigma_{\mathrm{ctrl}} = I_k$, and α=1, then (8) reduces to

$$D_{\mathrm{KL}}\big(p(\cdot;\theta)\|p(\cdot;\theta+\delta u)\big) \ge \frac{\delta^2}{2k}R(\theta) + o(\delta^2),$$

which corresponds to the baseline responsiveness floor adopted in the IEPI formulation.

Proposition 5 (Coarse-graining stability)

Let Y be induced from X by a Markov kernel $K(y|x)$ independent of θ. Then $I_Y(\theta) \preceq I_X(\theta)$, $\mathrm{tr}\,I_Y(\theta) \le \mathrm{tr}\,I_X(\theta)$. Thus aggregation cannot increase responsiveness [12]

Proof. Let $p_X(x;\theta)$ denote the model on X and $p_Y(y;\theta) = \sum K(y|x)p_X(x;\theta)$ the induced model on Y. Define the X-score $s_X(x;\theta) = \nabla_\theta \log p_X(x;\theta)$, so $I_X(\theta) = \mathbb{E}\big[s_X s_X^\top\big]$.

Since K does not depend on θ,

$$s_Y(y;\theta)=\nabla_\theta \log p_Y(y;\theta)=\frac{\sum_x K(y\mid x)\nabla_\theta p_X(x;\theta)}{p_Y(y;\theta)}=\mathbb{E}\left[s_X(X;\theta)\mid Y=y\right].$$

Therefore,

$$I_Y(\theta)=\mathbb{E}\left[s_Y s_Y^\top\right]=\mathbb{E}\left[\mathbb{E}[s_X\mid Y]\,\mathbb{E}[s_X\mid Y]^\top\right]\preceq\mathbb{E}\left[\mathbb{E}\left[s_X s_X^\top\mid Y\right]\right]=\mathbb{E}\left[s_X s_X^\top\right]=I_X(\theta),$$

where the Löwner inequality follows from the variance decomposition

$$\mathbb{E}\left[s_X s_X^\top\right]=\mathbb{E}\left[\mathrm{Cov}(s_X\mid Y)\right]+\mathbb{E}\left[\mathbb{E}[s_X\mid Y]\,\mathbb{E}[s_X\mid Y]^\top\right]\succeq\mathbb{E}\left[\mathbb{E}[s_X\mid Y]\,\mathbb{E}[s_X\mid Y]^\top\right].$$

Taking traces preserves the inequality, hence $\operatorname{tr} I_Y(\theta)\le \operatorname{tr} I_X(\theta)$.

IEPI feasibility region and penalty functional

Combining (3)–(4), define the viability region.

$$\mathcal{V}=\left\{\,p\in\Delta_{n-1}:H(p)\ge H^*_{\min},\,R(p)\ge\rho_{\min}\,\right\}.$$

To enforce full viability, introduce the hinge penalty

$$\Phi(p)=\left[H^*_{\min}-H(p)\right]_+ +\left[H(p)-H^*_{\max}\right]_+ +\left[\rho_{\min}-R(p)\right]_+,$$

where $[x]_+ = max(x,0)$ denotes the positive part. The upper-entropy ceiling $H^*_{\max}$ regulates operational viability but is excluded from convexity considerations. The convex component of the penalty is therefore

$$\Phi_{\mathrm{cvx}}(p)=\left[H^*_{\min}-H(p)\right]_+ +\left[\rho_{\min}-R(p)\right]_+.$$

Hinge-type penalty constructions of this form are widely used in convex optimization frameworks for constraint enforcement and real-time feasibility control [13].

Structural composition and stability

For each process fragment B_g, let $U(B_g)$ and $R(B_g)$ denote its respective entropy and responsiveness contributions. These quantities combine under standard workflow constructs as follows.

Sequential (AND) composition

Entropy and responsiveness accumulate additively:

$$U(\mathrm{SEQ})=\sum_g U(B_g),\qquad R(\mathrm{SEQ})=\sum_g R(B_g).$$

Exclusive choice (XOR) split

For branch probabilities $p = (p_g)$,

$$U(\mathrm{XOR})=H(p)+\sum_g p_g U(B_g),\qquad R(\mathrm{XOR})=1-\|p\|_2^2$$

Inclusive choice (OR) split

Under the same routing distribution p [1]:

$$U(\mathrm{OR})=H_2(p)+\sum_g p_g U(B_g),\qquad R(\mathrm{OR})=\sum_g p_g(1-p_g).$$

1 The original formulation of Jung et al. (2011) expresses the OR–split uncertainty as a sum of binary Shannon entropies $\sum h(p_g)$, where *h(p)=-plogp-(1-p)log(1-p)*. In the IEPI framework this is replaced by the collision–entropy surrogate $H_2(p)=-\log\sum p_g^2$, which ensures analytical compatibility with the entropy–responsiveness coupling *H≥-log(1-R)* and maintains coherence with Theorem 1. This substitution preserves the conceptual role of the OR construct while integrating it consistently into the IEPI viability formalism.

Loop construct

For continuation probability $q\in(0,1)$ and loop body B_{body},

$$U(\mathrm{LOOP})=H(q)+\frac{q}{1-q}H(B_{\mathrm{body}}),\qquad R(\mathrm{LOOP})=q(1-q).$$

These composition rules preserve both the entropy lower bound and the responsiveness floor, provided that all routing distributions satisfy $R\ge\rho_{\min}$.

Lemma 2 (Closure under composition)

Let each sub-block B_g (and in loop structures, the body B_{body}). If all components lie in $\mathcal{V}$ and each routing distribution satisfies $R\ge\rho_{\min}$, then SEQ/AND, XOR/OR, and admissible LOOP compositions remain within $\mathcal{V}$, where admissibility requires $q\in(0,1)$ with $q(1-q)\ge\rho_{\min}$ and $H(q)+\frac{q}{1-q}H(B_{\mathrm{body}})\ge H^*_{\min}$

These composition rules extend the canonical formulations of process-pattern entropy given in Table 2 of [10], expressed here in natural logarithmic units and generalized to include responsiveness terms. The LOOP expression corresponds to the asymptotic expectation of the finite-iteration form in [10], ensuring analytical consistency with the IEPI framework.

Existence and structural stability of IEPI

Proposition 6 (Existence and stability)

Let $n\ge 2$, $0<H^*_{\min}<\log n$, and $0\le\rho_{\min}<R_{\max}$ with $R_{\max}=1-1/n$. *Then* $\mathcal{V}=\left\{p\in\Delta_{n-1}:H(p)\ge H^*_{\min},R(p)\ge\rho_{\min}\right\}$ *is non-empty, convex, and compact. Moreover, under the composition rules of Section 3.7 (SEQ/AND, XOR, OR, LOOP),* $\mathcal{V}$ *is structurally stable: feasible components compose to feasible composites provided each routing distribution satisfies* $R\ge\rho_{\min}$ *and the LOOP parameters are admissible.*

Proof. Consider the simplex $\Delta_{n-1}=\{p\in\mathbb{R}^n_+:\sum_{i=1}^n p_i=1\}$ and define

$$H(p)=-\sum_{i=1}^n p_i\log p_i,\qquad R(p)=1-\sum_{i=1}^n p_i^2.$$

Fix thresholds $H^*_{\min}\in(0,\log n)$ and $\rho_{\min}\in\left[0,1-1/n\right)$.

Convexity and compactness: For $p_i > 0$, one has $\nabla^2 H(p)=-\mathrm{diag}(1/p_i)$, which is negative definite on every face of Δ_{n-1}; hence H is strictly concave. Since $p\mapsto-\|p\|_2^2$ is concave on $\mathbb{R}^n$, R is concave as well. Therefore, the superlevel sets $\{H\ge H_{\min^*}\}$ and are convex, and their intersection

$$\mathcal{V}=\left\{p\in\Delta_{n-1}:H(p)\ge H^*_{\min},R(p)\ge\rho_{\min}\right\}$$

is convex. Continuity of H and R, together with compactness of Δ_{n-1}, ensures that $\mathcal{V}$ is closed and compact.

Non-emptiness

Let $u=(1/n,\ldots,1/n)$. Then $H(u)=\log n>H^*_{\min}$ and $R(u)=1-1/n>\rho_{\min}$, implying $u\in\mathcal{V}$.

Structural stability under composition

Adopt the composition formulas of Section 3.7. Assume that each process fragment B_g satisfies

$$U(B_g)\ge H^*_{\min},\qquad R(B_g)\ge\rho_{\min},$$

and that each routing distribution obeys
Sequential and parallel (AND) composition obey additivity:

$$U(\mathrm{SEQ}) = \sum_g U(B_g), \qquad R(\mathrm{SEQ}) = \sum_g R(B_g).$$

Since each B_g already satisfies the feasibility conditions, the composite process remains viable.

For the exclusive-choice (XOR) construct,

$$U(\mathrm{XOR}) = H(p) + \sum_g p_g U(B_g), \qquad R(\mathrm{XOR}) = 1 - \| p \|_2^2,$$

so $U(\mathrm{XOR}) \geq H^*_{\min}$ and $R(\mathrm{XOR}) \geq \rho_{\min}$ when routing is feasible.

Similarly, the inclusive-choice (OR) structure satisfies

$$U(\mathrm{OR}) = H_2(p) + \sum_g p_g U(B_g),$$

$$R(\mathrm{OR}) = \sum_g p_g (1 - p_g) = 1 - \| p \|_2^2,$$

thus preserving feasibility under the same constraint.

For the loop construct with continuation probability $q \in (0,1)$:

$$U(\mathrm{LOOP}) = H(q) + \frac{q}{1-q} U(B_{\mathrm{body}}), \qquad R(\mathrm{LOOP}) = q(1-q).$$

Since $q(1-q) \leq 1/4$, viability requires $\rho_{\min} \leq 1/4$ and

$$q(1-q) = \rho_{\min}$$

Solving $q(1-q) = \rho_{\min}$ yields

$$q_\rho = \frac{1 + \sqrt{1 - 4\rho_{\min}}}{2} \in [1/2, 1),$$

ensuring $U(\mathrm{LOOP}) \geq H_{\min^*}$ or all $q \in [q_\rho, 1)$

Therefore, $\mathcal{V}$ is convex, compact, non-empty, and stable under all IEPI-preserving compositions: SEQ/AND, XOR, OR, and admissible LOOP constructs with $\rho_{min} \leq 1/4$. This concludes the proof.

Interpretation: the IEPI manifold

The set $\mathcal{V}$ defines the admissible region in the (H, R)–plane within which informational uncertainty and control responsiveness remain jointly sustainable. Entropy cannot fall below the responsiveness-induced lower bound, which would correspond to deterministic rigidity and loss of adaptive variability, nor can it exceed the divergence ceiling; an operational upper-entropy limit beyond,[2] which responsiveness becomes asymptotically ineffective. Within this feasible manifold, processes preserve both sensitivity to perturbations and stability under regulation, expressing the operational equilibrium that the IEPI framework formalizes. This equilibrium characterizes viable process behaviour and establishes the reference geometry for the structural viability analysis developed in the following section.

2 Here "deterministic rigidity" denotes the operational collapse of variability when entropy approaches its minimum value H=0, resulting in a fully predictable process configuration. It does not invoke "superdeterminism" in the quantum-mechanical sense, which refers to the absence of genuine stochastic freedom in underlying physical states (cf. [14], Section 3.7.1, pp. 44–45).

Illustrative IEPI Computation and Practical Implications

This section demonstrates IEPI viability assessment in representative control-flow structures. In contrast to entropy-only analyses, IEPI requires that both global uncertainty and local adaptive sensitivity remain within the informationally stable zone defined in (11), with the entropy–responsiveness coupling of Theorem 1 ensuring a quantitative linkage between these two stability requirements. Even at this preliminary (a-stage) implementation, the empirical consistency of IEPI across distinct control-flow configurations lends support to the theoretical and epistemological conclusions established in the broader literature on entropy-based process modelling, where informational symmetry and entropic balance were shown to underpin structural coherence and systemic viability in business process environments [15].

Primitive control–flow patterns

Let $U(\cdot)$ and $R(\cdot)$ denote, respectively, the entropy and responsiveness contributions of a process fragment. Following the formal definitions of uncertainty for canonical control–flow constructs presented in [10], the main primitives combine as follows.

For a routing distribution $r = (r_g)$, an exclusive–choice (XOR) split contributes

$$U(\mathrm{XOR}) = H(r), \qquad R(\mathrm{XOR}) = 1 - \| r \|_2^2,$$

reflecting informational uncertainty over the selected branch together with the categorical responsiveness of the router.

For an inclusive–choice (OR) split governed by the same routing vector r, viability depends on potential joint activation of fragments. Writing $h(p) = -p \log p - (1-p)\log(1-p)$ for the binary Shannon entropy, the entropy and responsiveness contributions are

$$U(\mathrm{OR}) = \sum_g h(r_g) + \sum_g r_g U(B_g), \qquad R(\mathrm{OR}) = \sum_g r_g (1 - r_g),$$

which are consistent with the per-branch toggling semantics in [10].

Loop constructs introduce iterative execution. With continuation probability q ∈ (0,1) and body fragment B_{body},

$$U(\mathrm{LOOP}) = H(q) + \frac{q}{1-q} U(B_{\mathrm{body}}), \qquad R(\mathrm{LOOP}) = q(1-q),$$

so responsiveness is maximized at $q = 1/2$ and vanishes as the process collapses into either immediate termination or infinite repetition.

Assessment proceeds by validating each control–flow primitive against the feasibility constraints defined in (11) and the entropy–responsiveness coupling of Theorem 1. This quantifies whether each routing configuration sustains both sufficient informational variability and controllable sensitivity to remain within the viable IEPI region (cf. [15]) for conceptual context).

Comparison of XOR and OR configurations under IEPI

Consider a process fragment with three downstream activities B_1, B_2, B_3 and routing probabilities r = (0.5,0.3,0.2). Two branching semantics are examined: in an exclusive–choice (XOR) split exactly one branch executes; in an inclusive–choice (OR) split, each branch may execute independently with the same marginal probabilities. For XOR,.

$$U(\mathrm{XOR}) = H(r), \qquad R(\mathrm{XOR}) = 1 - \| r \|_2^2,$$

and the coupling floor is $H_{\text{floor}} = -\log(1-R)$ For OR,

$$U(\mathrm{OR}) = H_2(r) + \sum_g r_g U(B_g), \qquad R(\mathrm{OR}) = \sum_g r_g (1 - r_g) = 1 - \| r \|_2^2,$$

so responsiveness coincides with XOR given the same router r, while entropy differs through the collision term $H_2(r)$ and the branch–internal contributions $U(Bg)$.

Regime A: moderate dispersion with nontrivial internal uncertainty.

With $r = (0.5,0.3,0.2)$ and representative internal contributions $(U(B_1),U(B_2),U(B_3)) = (0.2,0.3,0.1)$ (nats),

$$H(XOR) = 1.0296, \;\; R = 0.62, \;\; H_{floor} = 0.9676,$$

$$U(\mathrm{OR}) = H_2(r) + \sum_g r_g U(B_g) = 0.9676 + 0.21 = 1.1776.$$

Hence the IEPI margins H-H_floor are

$$\text{XOR margin} = 1.0296 - 0.9676 = 0.0620,$$

$$\text{OR margin} = 1.1776 - 0.9676 = 0.2100,$$

so OR maintains a larger viability buffer.

Regime B: skewed routing with negligible internal uncertainty.

With a highly imbalanced router r = (0.9,0.09,0.01) and U (Bg) ≈ 0,

$$H(\mathrm{XOR}) = 0.3576, \quad R = 0.1818, \quad H_{\text{floor}} = H_2(r) = 0.2006,$$

$$U(\mathrm{OR}) = H_2(r) = 0.2006$$

The resulting margins are

$$\text{XOR margin} = 0.3576 - 0.2006 = 0.1569,$$

$$\text{OR margin} = 0.2006 - 0.2006 = 0,$$

so XOR strictly dominates OR. This occurs because, with negligible branch–internal uncertainty, $U(OR)=H_2(r) \le H(r) = U(XOR)$, while both share the same responsiveness.

IEPI margins for XOR vs. OR under two regimes (entropy in nats).

Regime	Config.	H or U	R	Margin (H-H_{floor})
A: r = (0.5,0.3,0.2), U(B)=(0.2,0.3,0.1)	XOR	1.0296	0.62	+0.0620
	OR	1.1776	0.62	+0.2100
B: r=(0.9,0.09,0.01), U(B)≈0	XOR	0.3576	0.1818	+0.1569
	OR	0.2006	0.1818	0

Design implication.

For fixed router r, responsiveness is identical for XOR and OR. The deciding factor is the branch–internal uncertainty $\sum_g r_g U(B_g)$: when internal variability is material (Regime A) OR enjoys a larger IEPI margin; when it is negligible and routing is skewed (Regime B), XOR is preferable. This provides a principled criterion for selecting control–flow semantics under IEPI constraints.

Responsiveness–based sufficiency

From the entropy–responsiveness coupling established in Theorem 1 (cf. also [10] for the underlying entropy formalism), the inequality.

$$R(p) \ge 1 - e^{-H^*_{\min}} \Rightarrow H(p) \ge H^*_{\min}$$

provides a sufficient condition for process viability expressed purely in terms of responsiveness. This criterion allows verification of the entropy threshold through responsiveness alone, without requiring explicit estimation of the full probability distribution. Although not necessary in general, it is often operationally preferable in practice, since responsiveness can be empirically approximated from control variability or execution dispersion metrics even when detailed routing probabilities are unavailable (cf. [15]).

Numerical illustration

Consider the router $r = (0.2,0.3,0.5)$. Then

$$H = 1.0296, \quad R = 0.62, \quad H_{\text{floor}} = -\log(1-R) = 0.9676.$$

Since $H > H_{floor}$, the configuration lies within the IEPI feasible region.

To compare alternative semantics, consider a binary decision context.

Table 2: *Parameter values for XOR (Model A) and OR (Model B) configurations.*

Config.	r	H	R	H_{floor}
A (XOR)	(0.7,0.3)	0.6109	0.42	0.5447
B (OR)	(0.7,0.3)	$h(0.7) + h(0.3) = 1.2217$	0.42	0.5447

Models A and B: exclusive–choice vs. inclusive–choice

Here $h(p) = -p\log p - (1-p)\log(1-p)$ denotes the binary Shannon entropy in natural units (nats).

Interpretation.

Under the adopted IEPI definitions, XOR and OR exhibit identical responsiveness for the same routing marginals and therefore share the same coupling floor. The OR configuration, however, yields greater entropy due to the additive contribution of two binary toggles, producing a higher IEPI margin H-H_floor and a correspondingly broader viability buffer. Within the IEPI framework, this margin represents the informational headroom that ensures adaptive control remains effective while avoiding rigidity at the lower bound. Accordingly, when both constructs satisfy the responsiveness threshold, the configuration with the larger IEPI margin, namely the OR split, is deemed more viable and resilient in sustaining balanced process dynamics. Conversely, when branch–internal uncertainty is negligible and routing probabilities are strongly skewed, the XOR construct becomes preferable, as its lower entropy mitigates proximity to the divergence ceiling and preserves controllability. Hence, IEPI evaluation prescribes OR semantics for moderately dispersed variability contexts and XOR semantics for sharply polarized, low–uncertainty regimes.

Controlled responsiveness and Fisher-informed redesign

When only selected variability directions permit intervention, responsiveness may be evaluated relative to a control metric $\Sigma_{\text{ctrl}} \succeq 0$ as

$$R_\Sigma(p) = \mathrm{tr}\big(I(\theta)\Sigma_{\mathrm{ctrl}}\big),$$

where $I(\theta)$ denotes the Fisher information matrix of the parameterized process distribution [6]. For categorical or softmax-based routers, $I(\theta) = diag(p) - pp^\top$ and $R_\Sigma(p)$ quantifies the cumulative control leverage over the accessible subspace defined by Σ_{ctrl}. A guaranteed floor $R_\Sigma(p) \geq \rho^\Sigma_{\min}$ then implies

$$H(p) \geq -\log\big(1-\rho^\Sigma_{\min}\big),$$

ensuring informational viability even when intrinsic dispersion is constrained. Fisher information thus measures how targeted control capacity locally enlarges the feasible region of the IEPI manifold by reinforcing responsiveness in controllable subspaces.

Illustrative computation

Consider the three-way router $r = (0.2, 0.3, 0.5)$, for which

$$I(\theta) = \mathrm{diag}(r) - rr^\top, \qquad R(p) = 1 - \|r\|_2^2 = 0.62,$$
$$H_{\mathrm{floor}} = -\log(1-R) = 0.9676.$$

Several control scenarios yield distinct controlled-responsiveness values:

$$\Sigma_{\mathrm{ctrl}} = I_3 \quad\Rightarrow\quad R_\Sigma = 0.62,\ H^\Sigma_{\mathrm{floor}} = 0.9676;$$
$$[4pt]\Sigma_{\mathrm{ctrl}} = \mathrm{diag}(1,1,0) \quad\Rightarrow\quad R_\Sigma = 0.37,\ H^\Sigma_{\mathrm{floor}} = 0.4620;$$
$$[4pt]\Sigma_{\mathrm{ctrl}} = \mathrm{diag}(0,0,1) \quad\Rightarrow\quad R_\Sigma = 0.25,\ H^\Sigma_{\mathrm{floor}} = 0.2877.$$

Restricting control to fewer directions decreases R_Σ and lowers the entropy floor indicating reduced guaranteed adaptability.

Interpretation.

IEPI distinguishes intrinsic uncertainty (quantified by H) from controllable adaptability (quantified by R_Σ). The quantity R_Σ captures the responsiveness that can be realized through effective intervention rather than the total latent variability of the process. Consequently, Fisher-informed redesign identifies which parameters or substructures most enhance viability by maximizing R_Σ within the feasible manifold. Increasing R_Σ expands the entropy floor thereby broadening the viable operational region without necessarily increasing overall entropy. This shift from maximizing H to optimizing controllable responsiveness marks the transition from passive assessment to active, data-informed process steering within the IEPI framework.

Finite-sample certification of IEPI viability

When routing probabilities are estimated from empirical counts $c = (c_1, \ldots, c_n)$ under a Dirichlet prior α [16], the posterior mean estimate is

$$\hat{r}_i = \frac{\alpha_i + c_i}{\sum_j(\alpha_j + c_j)}, \qquad \hat{H} = -\sum_i \hat{r}_i \log \hat{r}_i, \qquad \hat{R} = 1 - \sum_i \hat{r}_i^2.$$

These posterior quantities serve as finite-sample analogues of their population counterparts.

Credible intervals for $\hat{R}$ yield probabilistic certification of IEPI viability. If the lower credible bound $\underline{R}$ satisfies

$$\underline{R} \geq \rho_{\min},$$

then the entropy–responsiveness coupling ensures

$$H \geq -\log(1-\underline{R}) \geq H^*_{\min}$$

with posterior credibility (1 - β), for confidence level (1 - β) of the credible region.

The diagnostic implications of observed outcomes are summarised in Table 3.

Condition	Interpretation	Design response
$H < H^*_{\min}$	Deterministic collapse	Expand controllable branching
$H > H^*_{\max}$	Stochastic instability	Reduce or rebalance concurrency
$R < \rho_{\min}$	Weak controllability	Enhance responsiveness via Σ_{ctrl}
Near bounds	Marginal viability	Tune parameters to restore margin
All constraints met	Stable regime	Maintain with periodic checks

These diagnostics provide actionable guidance for structural redesign and monitoring under sampling uncertainty. They allow decision-makers to certify IEPI viability probabilistically, rather than deterministically, ensuring that process adjustments are both data-informed and statistically defensible.

Epistemic and design interpretation

The IEPI framework extends beyond quantitative certification to provide an epistemic and structural interpretation of process viability. Because modifications in routing distributions, control leverage, or feedback topology induce predictable displacements in the (H, R) coordinate space, the IEPI manifold serves as a geometrically interpretable map of operational trade-offs. Entropy H captures intrinsic informational diversity, representing the potential for adaptation, while responsiveness R (or its controlled counterpart R_Σ) quantifies actionable adaptability through effective sensitivity to intervention. The coupling relation between these quantities defines the viability band within which processes remain informationally stable.

In design terms, maintaining $H^*_{\min} \leq H \leq H^*_{\max}$ and $R \geq \rho_{\min}$ ensures that uncertainty and controllability are jointly sustainable. Entropy must be sufficiently high to avoid deterministic rigidity, yet bounded to prevent stochastic drift. Responsiveness, in turn, must exceed its minimum floor, either empirically or as derived from Fisher-informed estimates, to guarantee that adaptive mechanisms remain effective even under partial controllability. Finite-sample certification further allows these properties to be validated probabilistically from empirical traces, ensuring that IEPI assessment remains compatible with real operational data.

Accordingly, IEPI provides both a quantitative diagnostic and a design-oriented epistemology. It offers a unified view of process structure in which informational entropy and control responsiveness jointly define sustainable behaviour under uncertainty. This perspective supports iterative, evidence-driven redesign of processes, maintaining viability within the feasible manifold while balancing flexibility with stability.

Conclusion and Outlook

This work develops the *Information Entropy Performance Indicator* (IEPI) as a discrete informational framework for assessing the viability of uncertain operational processes. Beginning from Shannon entropy on the probability simplex, the analysis treats control-flow structures as uncertainty-generating blocks whose informational dispersion can be quantified and bounded. The introduction of functional entropy limits defines an operational spectrum between deterministic rigidity and stochastic instability, within which sustainable behaviour is possible.

Responsiveness, defined through the trace of the Fisher information, provides a complementary measure of local sensitivity to perturbation. The entropy–responsiveness coupling inequality demonstrates that a minimal degree of uncertainty is mathematically required to preserve adaptability, establishing a bidirectional relationship between dispersion and controllability. Together they delineate an informationally stable region, the domain in which process performance remains both predictable and steerable.

The geometry extends naturally to global comparisons. Relative entropy $D_{\mathrm{KL}}(p \| q)$ furnishes a principled measure of deviation from reference behaviour, and continuity bounds ensure that viability persists under controlled statistical drift. Moreover, the closest IEPI-feasible redesign can be formulated as

$$\min_{\tilde{p} \in \mathcal{V}} D_{\mathrm{KL}}(\tilde{p} \| p),$$

revealing a convex relationship between structural change and informational cost. In this way, Shannon entropy, Fisher information, and Kullback–Leibler divergence form a coherent analytic chain linking global uncertainty, local sensitivity, and controlled adaptation.

The results presented here establish the existence, stability, and practical computability of IEPI, but they constitute a starting point rather than a final theory. Further research should extend the framework to semi-Markov and continuous-time formulations, derive robust bounds under observation noise, and examine structural optimisation in high-dimensional state spaces. By connecting discrete uncertainty analysis with information geometry, IEPI provides a quantitative basis for understanding how systems maintain viable operation under uncertainty. The continued refinement of this approach is expected to benefit from collaboration across applied mathematics, systems engineering, and information theory.

Key contributions

- The introduction of IEPI establishes a new analytical perspective on process viability grounded in discrete information theory and information geometry. The principal contributions of this work are:
- A mathematically rigorous formulation of viability through a dual coupling of Shannon entropy and Fisher information, including analytical bounds on the feasible region.
- Proof of convexity, non-emptiness, and compositional stability of the viability set for structured processes, ensuring scalability to complex control-flow configurations.
- Introduction of responsiveness-based entropy guarantees, curvature bounds, and coarse-graining stability results connecting local and global information geometry.
- Development of operational assessment techniques, including Fisher-informed redesign and finite-sample viability certification.
- These results form the theoretical foundation of an entropy-regulated information-performance metric. Future work will expand the framework and explore domain-specific applications where uncertainty and controllability must be jointly managed to sustain adaptive and resilient behaviour.

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Author Contributions

Informed consent was obtained from all participants who have participated in the study.

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