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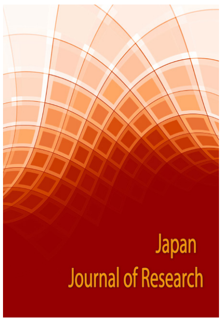

Spin-Electromagnetics

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Abstract

The spin of electrons is studied (Spintronics) and applied to, for example, quantum computing. In this article, utilizing UMFT (Universal Mathematical Field Theory), we establish the Spin-Electromagnetics (Spin-EM) from the Coulomb's law and the spin of electric source. Spin-EM predicts several new effects including the Spin-Lorentz-type force, and Spin-Lagrangian-Lorentz-type force. When experimentally confirmed, the Spin-force would be a NEW FORCE. The UMFT has been established and utilized to derive the Maxwell electromagnetics (Maxwell-EM) and Maxwell-type gravity (Gravito-EM). UMFT shows that mathematical identities lead to physical dualities, such as the duality between Maxwell-EM, Gravito-EM, and Spin-EM. The well-known-concepts, effects, and phenomena of Maxwell-EM may be directly converted to that of Spin-EM. The Maxwell-EM, Gravito-EM, and Spin-EM are all derived from UMFT and thus, they are dual to each other, and can be unified in the frame of UMFT.

Introduction

Historically, Maxwell equations were established based on series experiments, except the introduction of the displacement current. The Coulomb's law states that a rest e-charge Q_e induces a static vector electric field **E**. The created electric field is determined by only one quantity, e-charge. A heuristic phenomenon is that when an e-charge uniformly moves, it creates not only the electric field but a magnetic field. The magnetic field is determined by three quantities, e-charge, electric field and velocity. In classical EM theory, there are two main states of motion of an e-charge: (1) static and (2) uniform motion.

Now we ask a question: Does the spin of a charge create new field/force.

- In Section 2, we review **Universal Mathematical Field Theory (UMFT)**. To mathematically derive field equations of a spin-charge.
- In Section 3, **Spin-Electromagnetics (Spin-EM)** is established by utilizing UMFT, Coulomb's law, e-charge, and spin $\mathbf{S}_e$ of the e-charge. Spin-EM provides classical foundations of variety phenomena of spintronics, and predicts new phenomena due to the spin of an e-charge.

Table 1. *Fields of e-charge*

Source	Fields	Force	Nature of Field
Static-e-charge Q_e	$\nabla\cdot\mathbf{E} = Q_e$	$q_e\,\boldsymbol{E}$	vector field
Moving-e-charge $Q_e\mathbf{v}$	$\nabla\cdot\mathbf{E} = Q_e$ $\nabla\times\mathbf{B} = Q_e\,\mathbf{v} + \partial\mathbf{E}/\partial t$ $\nabla\times\mathbf{E} = -\partial\mathbf{B}/\partial t$ $\nabla\cdot\mathbf{B} = 0$	$q_e\,\boldsymbol{E} +$ $+\,q_e\,\mathbf{v}\times\mathbf{B}$	First order axial vector field
Spin-e-charge $Q_e\,\mathbf{S}_e$	$\nabla\cdot\mathbf{E}_s$ $\nabla\times\mathbf{B}_s$ $\nabla\times\mathbf{E}_s$ $\nabla\cdot\mathbf{B}_s$	$Q_e\,\boldsymbol{E}_s +$ $+\,Q_e\,\mathbf{v}\times\mathbf{B}_s$ $+Q_e\,\mathbf{S}_e\times\mathbf{B}$ $+Q_e\,\mathbf{S}_e\times\mathbf{B}_s$	Second order axial vector field

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Spin-EM predicts several new effects, the Spin-Lorentz-type force, and Spin-Lagrangian-Lorentz-type force. When experimentally confirmed, the Spin-forces would be NEW FORCES.

Spin-EM may be applied to different areas, for example, plasma/Tokamak.

The Maxwell-EM and Spin-EM are all derived from UMFT and thus, they are dual to each other, and can be unified in the frame of UMFT.

In Appendix, we derive Spin-gravity from the Newton's law and the spin of the gravity-g-charge (mass).

Universal Mathematical Field Theory (UMFT)

Motivation

There are three states of motion of a charge: (1) stationary; (2) uniform motion; (3) Spin. Utilizing UMFT, we have shown that the velocity of e-charge and g-charge produces Maxwell-EM [1] and Gravito-EM [2], respectively. UMFT provides the mathematic origin of dualities between different physic fields derived from it, such as the duality between Maxwell-EM and Gravito-EM.

In this article, we establish the Spin-EM (and Spin-gravity in Appendix). Duality is a powerful tool to find intrinsic similarities between apparently different phenomena, and predict new effects. "*It turns out that most of the important concepts and theories of physics can be unified and understood by their common attribute of duality*" (Damian P Hampshire).

Note: Spin-EM needs to be further tested experimentally.

UMFT: Vector Analysis Identity

To establish UMFT, we utilize mathematical identities that connecting divergences of either a vector or an axial vector with curls of induced axial vectors. For this aim, the following vector analysis identities are the most significant foundation of UMFT,

$$\nabla\times(\mathbf{R}\times\mathbf{T})=\mathbf{R}(\nabla\cdot\mathbf{T})-\mathbf{T}(\nabla\cdot\mathbf{R})+(\mathbf{T}\cdot\nabla)\mathbf{R}-(\mathbf{R}\cdot\nabla)\mathbf{T}, \tag{2.1}$$

$$\nabla\cdot(\mathbf{R}\times\mathbf{T})=(\nabla\times\mathbf{R})\cdot\mathbf{T}-\mathbf{R}\cdot(\nabla\times\mathbf{T}) \tag{2.2}$$

Equation (2.1) indicates that the combination of gradient and divergence of two arbitrary vectors including axial vector, **R** and **T**, induces inevitably an axial vector (**R** × **T**). One of two terms, either (∇·**T**) or (∇·**R**), represents fundamental inverse-square laws, and introduce a "charge". Now there are three quantities, R, T and "charge" in Equation (2.1).

It is useful to write Equation (2.1) in a different but equivalent form. By using the mathematical identity,

$$(\mathbf{T}\cdot\nabla)\mathbf{R}=\nabla(\mathbf{R}\cdot\mathbf{T})-(\mathbf{R}\cdot\nabla)\mathbf{T}-\mathbf{R}\times(\nabla\times\mathbf{T})-\mathbf{T}\times(\nabla\times\mathbf{R})$$

Equation (2.1) can be rewritten as an identity,

$$\nabla\times(\mathbf{R}\times\mathbf{T})=\mathbf{R}(\nabla\cdot\mathbf{T})-\mathbf{T}(\nabla\cdot\mathbf{R})+2(\mathbf{T}\cdot\nabla)\mathbf{R}-\nabla(\mathbf{R}\cdot\mathbf{T})+\mathbf{R}\times(\nabla\times\mathbf{T})+\mathbf{T}\times(\nabla\times\mathbf{R}) \tag{2.3}$$

Based on mathematical vector identities, we establish self-consistent UMFT (Equation (2.1) to Equation (2.3) that universally describes physical fields induced by the different motions of a source.

UMFT for Constant Spin of Charges

UMFT Related with Spin of Charges

We apply UMFT to spin.

(1). Maxwell-type equations.

The combination of the inverse-square law and the spin S of charges must induce axial vector fields. Let

R ≡ **S** = constant, then

$$\mathbf{T}(\nabla\cdot\mathbf{S})=(\mathbf{T}\cdot\nabla)\mathbf{S}=(\nabla\times\mathbf{S})=0 \tag{2.4}$$

Substituting Equation (2.4) into Equation (2.1) to Equation (2.3), respectively, we obtain Maxwell-type equations,

$$\nabla\times(\mathbf{S}\times\mathbf{T})=\mathbf{S}(\nabla\cdot\mathbf{T})-(\mathbf{S}\cdot\nabla)\mathbf{T} \tag{2.5}$$

$$\nabla\cdot(\mathbf{S}\times\mathbf{T})=-\mathbf{S}\cdot(\nabla\times\mathbf{T}) \tag{2.6}$$

$$\nabla\times(\mathbf{S}\times\mathbf{T})=\mathbf{S}(\nabla\cdot\mathbf{T})-\nabla(\mathbf{S}\cdot\mathbf{T})+\mathbf{S}\times(\nabla\times\mathbf{T}) \tag{2.7}$$

Note: physical quantities **R** representing are not limited to spin.

(2). Poynting theorem.

Let both "**R**" and "**T**" are fields, for example, **R = E** and **T = B**, Equation (2.2) gives "Poynting theorem",

$$\nabla\cdot(\mathbf{E}\times\mathbf{B})=(\nabla\times\mathbf{E})\cdot\mathbf{B}-\mathbf{E}\cdot(\nabla\times\mathbf{B})=-\frac{1}{2}\partial\left(\mathbf{B}^2+\mathbf{E}^2\right)/\partial t-\mathbf{E}\cdot\mathbf{J} \tag{2.8}$$

Equation (2.8) validates Equation (2.2).

In Section 3, we utilize Equation (2.5) to Equation (2.8) to derive the Maxwell-type equations for the fields induced by the spin of e-charges, referred it as Spin-EM.

Spin-EM

UMFT is the math theory and thus can be applied to suitable physical situations. Applying UMFT, we have re-derived Maxwell equations, which justifies UMFT and shows that the experiments-based Maxwell equations have their mathematical origin. In Section 3, we establish Spin-Electromagnetics (referred as Spin-EM). The Spin-EM is powerful and fruitful in the perspective of fundamental physics: (1) predicts new forces, the Lorentz-type-spin-force and Lagrangian-Lorentz-type-spin-force, which are spin related force; (2) UMFT shows that mathematical identities lead to physical dualities including duality between Maxwell-EM and Spin-EM.

By applying UMFT to the combination of spin and EM, we establish Spin-EM below.

Definitions of Spin-electric Field E_s and Spin-magnetic Field B_s

First let's consider a spin-e-charge, such as an electron, characterized by electric charge Q_e and spin $\mathbf{S}_e$. The e-charge Q_e produces a Coulomb field, $\nabla\cdot\mathbf{E} = Q_e$.

We propose that the spin $\mathbf{S}_e$ of the e-charge Q_e induces axial vector fields:

A static spin-e-charge Q_e with spin $\mathbf{S}_e$ produces an electric field E and a spin-magnetic field $\mathbf{B}_s$,

$$\mathbf{B}_s\equiv\mathbf{S}_e\times\mathbf{E}, \tag{3.1a}$$

A spin-e-charge Q_e moving with velocity **v** produces an electric field **E**, a magnetic field **B**, a spin-magnetic field $\mathbf{B}_s$ (polar vector), and a spin-electric field $\mathbf{E}_s$ (Axial vector), where,

$$\mathbf{B}_s\equiv\mathbf{S}_e\times\mathbf{E}, \tag{3.1b}$$

$$\mathbf{E}_s\equiv-\mathbf{S}_e\times\mathbf{B}. \tag{3.2}$$

Subscript "s" indicates the quantity related to spin. The $\mathbf{B}_s$ and $\mathbf{E}_s$ are type-2 dual to the **B** and **E** fields, respectively. The definitions, Equation (3.1) and Equation (3.2), predict a phenomenon: by interacting with the electric field **E** and the magnetic field **B**, the spin of e-charge produces a spin-magnetic field $\mathbf{B}_s$ and a spin-electric field $\mathbf{E}_s$, respectively.

Note: (1) the definitions of $\mathbf{B}_s$ and $\mathbf{E}_s$ are conceptually different from that of Rashba-induced-Spin-EM.

(2) we do not consider the higher-order-terms of $\mathbf{S}_e \times \mathbf{E}_s$ and $\mathbf{S}_e \times \mathbf{B}_s$.

Spin-EM Equation

Spin-EM can be derived from the UMFT formular, Equation (2.5) to Equation (2.7). To applying Equation (2.5), Equation (2.6) and Equation (2.7) to the spin-e-charge, let $\mathbf{R} \equiv \mathbf{S}_e$ = conctant; $\mathbf{S}_e$ is the spin of the spin-e-charge.

Let **T** = **E** (the electric field) and **T** = **B** (the magnetic field), respectively. Then substituting Equation (3.1) and Equation (3.2) into Equation (2.5) to Equation (2.7) respectively, we obtain:

$$\nabla \times \mathbf{B}_s = \nabla \times (\mathbf{S}_e \times \mathbf{E}) = \mathbf{S}_e (\nabla \cdot \mathbf{E}) - (\mathbf{S}_e \cdot \nabla)\mathbf{E} \quad (3.3a)$$

$$\nabla \times \mathbf{B}_s = \mathbf{S}_e (\nabla \cdot \mathbf{E}) - \nabla (\mathbf{S}_e \cdot \mathbf{E}) + \mathbf{S}_e \times (\nabla \times \mathbf{E}) \quad (3.3b)$$

$$\nabla \cdot \mathbf{B}_s = \nabla \cdot (\mathbf{S}_e \times \mathbf{E}) = -\mathbf{S}_e \cdot (\nabla \times \mathbf{E}) \neq 0. \quad (3.4)$$

$$\nabla \times \mathbf{E}_s = \nabla \times (-\mathbf{S}_e \times \mathbf{B}) = -\mathbf{S}_e (\nabla \cdot \mathbf{B}) + (\mathbf{S}_e \cdot \nabla)\mathbf{B} \quad (3.5a)$$

$$\nabla \times \mathbf{E}_s = -\mathbf{S}_e (\nabla \cdot \mathbf{B}) + \nabla (\mathbf{S}_e \cdot \mathbf{B}) - \mathbf{S}_e \times (\nabla \times \mathbf{B}) \quad (3.5a)$$

$$\nabla \cdot \mathbf{E}_s = \nabla \cdot (-\mathbf{S}_e \times \mathbf{B}) = \mathbf{S}_e \cdot (\nabla \times \mathbf{B}) \quad (3.6)$$

Note: it is interesting that in Maxwell-EM, we have $\nabla \cdot \mathbf{B} = 0$.

Equation (3.3) and Equation (3.4) are dual to Equation (3.5) and Equation (3.6), respectively, under the transformation:

$\mathbf{E} \leftrightarrow -\mathbf{B}$ and $\mathbf{E}_s \leftrightarrow \mathbf{B}_s$

Substituting Maxwell equations into Equation (3.3) to Equation (3.6), we have Spin-EM equations:

$$\nabla \times (\mathbf{B}_s) = Q_e \mathbf{S}_e - (\mathbf{S}_e \cdot \nabla)\mathbf{E}, \quad (3.7a)$$

$$\nabla \times (\mathbf{B}_s) = Q_e \mathbf{S}_e + \frac{\partial}{\partial t} \mathbf{E}_e - \nabla (\mathbf{S}_e \cdot \mathbf{E}) \quad (3.7b)$$

$$\nabla \cdot (\mathbf{B}_s) = \frac{\partial}{\partial t} (\mathbf{S}_e \cdot \mathbf{B}) \quad (3.8)$$

$$\nabla \times \mathbf{E}_s = (\mathbf{S}_e \cdot \nabla)\mathbf{B} \quad (3.9a)$$

$$\aleph = \mathbf{E}_s \aleph \quad \frac{\partial \mathbf{B}_s}{\partial t} \quad Q_e \mathbf{S}_e \quad \mathbf{v} \quad (\mathbf{S}_e \;\; \mathbf{B}) \quad (3.9b)$$

$$\nabla \cdot \mathbf{E}_s = \mathbf{S}_e \cdot (\nabla \times \mathbf{B}) = Q_e \mathbf{S}_e \cdot \mathbf{v} + \frac{\partial}{\partial t} (\mathbf{S} \cdot \mathbf{E}) \quad (3.10)$$

Equation (3.7) to Equation (3.10) are the complete set of Spin-EM equations for constant spin.

Equation (3.7) and Equation (3.9) are the Ampere-type equation of spin and Faraday-type equations of spin, respectively.

Note: the spin $\mathbf{S}_e$ of an e-charge generates the Spin-EM field, which is conceptually different from that of Rashba-induced-Spin-EM.

Poynting-type theorem of Spin-EM

Equation (2.2), $\nabla \cdot (\mathbf{R} \times \mathbf{T}) = (\nabla \times \mathbf{R}) \cdot \mathbf{T} - \mathbf{R} \cdot (\nabla \times \mathbf{T})$, shows the Poynting-type theorem of Spin-EM:

(*0) Let **R** = **E** and **T** = **B**, Equation (2.2) gives the standard Poynting theorem:

$$\nabla \cdot (\mathbf{E} \times \mathbf{B}) = (\nabla \times \mathbf{E}) \cdot \mathbf{B} - \mathbf{E} \cdot (\nabla \times \mathbf{B})$$

Which shows that Equation (2.2) is applicable to Poynting theorem.

(*1) Let $\mathbf{R} = \mathbf{E}_s$ and $\mathbf{T} = \mathbf{B}_s$, Equation (2.2) gives the spin-Poynting theorem (a):

$$\nabla \cdot (\mathbf{E}_s \times \mathbf{B}_s) = (\nabla \times \mathbf{E}_s) \cdot \mathbf{B}_s - \mathbf{E}_s \cdot (\nabla \times \mathbf{B}_s) \quad (3.11a)$$

(*2) Let $\mathbf{R} = \mathbf{E}_s$ and **T** = **B,** Equation (2.2) gives the spin-Poynting theorem (b):

$$\nabla \cdot (\mathbf{E}_s \times \mathbf{B}) = (\nabla \times \mathbf{E}_s) \cdot \mathbf{B} - \mathbf{E}_s \cdot (\nabla \times \mathbf{B}) \quad (3.11b)$$

(*3) Let **R** = **E** and $\mathbf{T} = \mathbf{B}_s$, Equation (2.2) gives the spin-Poynting theorem (c):

$$\nabla \cdot (\mathbf{E} \times \mathbf{B}_s) = (\nabla \times \mathbf{E}) \cdot \mathbf{B}_s - \mathbf{E} \cdot (\nabla \times \mathbf{B}_s) \quad (3.11c)$$

Table 2 summarizes the Spin-EM equations including the Poynting-type theorem.

***Table 2**. Spin-EM Equations*

EM: Maxwell Equations	$\nabla \cdot \mathbf{E} = Q_e$	$\nabla \cdot \mathbf{B} = 0$
	$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$	$\nabla \times \mathbf{B} = Q_e \mathbf{v} + \partial \mathbf{E}/\partial t$
	$\nabla \cdot (\mathbf{E} \times \mathbf{B}) = (\nabla \times \mathbf{E}) \cdot \mathbf{B} - \mathbf{E} \cdot (\nabla \times \mathbf{B})$	
Spin-EM: Definition	Spin-electric field $\mathbf{E}_s \equiv -\mathbf{S}_e \times \mathbf{B}$	Spin-magnetic field $\mathbf{B}_s \equiv \mathbf{S}_e \times \mathbf{E}$
Spin-EM: Field Equations	$\nabla \cdot \mathbf{E}_s = -\nabla \cdot (-\mathbf{S}_e \times \mathbf{B})$ $= -(\nabla \times \mathbf{S}_e) \cdot \mathbf{B} + \mathbf{S}_e \cdot (\nabla \times \mathbf{B})$ $= \mathbf{S}_e \cdot (\nabla \times \mathbf{B})$	$\nabla \cdot \mathbf{B}_s = \nabla \cdot (\mathbf{S}_e \times \mathbf{E})$ $= (\nabla \times \mathbf{S}_e) \cdot \mathbf{E} - \mathbf{S}_e \cdot (\nabla \times \mathbf{E})$ $= -\mathbf{S}_e \cdot (\nabla \times \mathbf{E})$
	$\nabla \times \mathbf{E}_s = \nabla \times (\nabla - \mathbf{S}_e \times \mathbf{B})$ $= -(\nabla \cdot \mathbf{B})\, \mathbf{S}_e + (\mathbf{S}_e \cdot \nabla)\, \mathbf{B}$ $\nabla \times \mathbf{E}_s =$ $= -\mathbf{S}_e \times (\nabla \times \mathrm{B}) + \nabla (\mathbf{S}_e \cdot \mathbf{B})$	$\nabla \times \mathbf{B}_s = \nabla \times (\mathbf{S}_e \times \mathbf{E})$ $= (\nabla \cdot \mathbf{E})\, \mathbf{S}_e - (\mathbf{S}_e \cdot \nabla)\, \mathbf{E}$ $\nabla \times \mathbf{B}_s =$ $= Q_e \mathbf{S}_e + \mathbf{S}_e \times (\nabla \times \mathbf{E}) - \nabla (\mathbf{S}_e \cdot \mathbf{E})$
Spin-EM: Poynting-type theorem	$\nabla \cdot (\mathbf{E}_s \times \mathbf{B}_s) = (\nabla \times \mathbf{E}_s) \cdot \mathbf{B}_s - \mathbf{E}_s \cdot (\nabla \times \mathbf{B}_s)$ $\nabla \cdot (\mathbf{E}_s \times \mathbf{B}) = (\nabla \times \mathbf{E}_s) \cdot \mathrm{B} - \mathbf{E}_s \cdot (\nabla \times \mathbf{B})$ $\nabla \cdot (\mathbf{E} \times \mathbf{B}_s) = (\nabla \times \mathrm{E}) \cdot \mathbf{B}_s - \mathbf{E} \cdot (\nabla \times \mathbf{B}_s)$	

Equations of Continuity of Spin Currents

By analogy to "electric current", let's define the "spin-magnetic-current $\mathbf{j}_{s\text{-}B}$" and the "spin-electric-current $\mathbf{j}_{s\text{-}E}$", which induce the spin-magnetic field $\mathbf{B}_s$ and the spin-electric field $\mathbf{E}_s$ respectively, as

$$\mathbf{j}_{s\text{-}B} \equiv \mathbf{S}_e (\nabla \cdot \mathbf{E}) - \nabla (\mathbf{S}_e \cdot \mathbf{E}), \quad (3.12)$$

$$\mathbf{j}_{s\text{-}E} \equiv \mathbf{S}_e (\nabla \cdot \mathbf{B}) - \nabla (\mathbf{S}_e \cdot \mathbf{B}) + \mathbf{S}_e \times (Q_e \mathbf{v}) \quad (3.13)$$

Equation (3.7b) and Equation (3.9b) become respectively,

$$\nabla \times \mathbf{B}_s = \mathbf{j}_{s\text{-}B} + \frac{\partial \mathbf{E}_s}{\partial t} \quad (3.14)$$

$$\nabla \times \mathbf{E}_s = -\mathbf{j}_{s\text{-}E} - \frac{\partial \mathbf{B}_s}{\partial t} \quad (3.15)$$

Scalar and Vector Potentials

By analogy to EM, defining spin-scalar-potential, φ_s, spin-vector-potential, $\mathbf{A}_s$, and gauge condition, as,

$$\mathbf{E}_s \equiv -\nabla \varphi_s - \frac{\partial \mathbf{A}_s}{\partial t}, \quad (3.16)$$

$$\mathbf{B}_s \equiv \nabla \times \mathbf{A}_s \quad (3.17)$$

$$\nabla \cdot \mathbf{A}_s + \frac{\partial}{\partial t} \varphi_s = 0. \quad (3.18)$$

Under the gauge transformation,

$$\mathbf{A}_s \to \mathbf{A}_s + \nabla\Lambda_s \qquad \varphi_s \to \varphi_s - \frac{\partial \Lambda_s}{\partial t}, \tag{3.19}$$

the spin-electric and spin-magnetic fields, $\mathbf{E}_s$ and $\mathbf{B}_s$, are invariant.

Spin Wave

Substituting Equation (3.16) to Equation (3.18) into Equation (3.14) and Equation (3.15) respectively, we have spin waves:

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\mathbf{A}_s = \mathbf{S}_e Q \ - \nabla\left(\mathbf{S}_e \cdot \mathbf{E}\right) \tag{3.20}$$

$$\left(\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\varphi_s\right) = \mathbf{S}_e \cdot (Q_e \mathbf{v}) + \frac{\partial}{\partial t}(\mathbf{S}_e \cdot \mathbf{E}) \tag{3.21}$$

Remark: The propagation speed of spin wave is to be determined experimentally.

Classical-Spin-EM vs. Quantum-Spin-EM

Let's study the similarity and difference between Classical-Spin-EM (C-Spin-EM) and Quantum-Spin-EM (Q-Spin-EM). To convert to quantum theory, we need to introduce the concept of phase,

$$\oint \mathbf{E}_s \cdot dl = -\frac{\partial}{\partial t}\oint \mathbf{A}_s \cdot \mathbf{dl} \tag{3.22}$$

$$\iint B_s \cdot \mathbf{ds} = \oint \mathbf{A}_s \cdot \mathbf{dl} \tag{3.23}$$

$$\oint \mathbf{A}_s \cdot \mathbf{dl} \equiv \phi_s \tag{3.24}$$

then have

$$\Phi_s = \iint \mathbf{B}_s \cdot \mathbf{ds}, \qquad \Phi_s = -\oint \mathbf{E}_s \cdot \mathbf{dl} \tag{3.25}$$

Defining Φ_s as the phase, $e^{i\Phi_s}$. Spin-electric field $\mathbf{E}_s$ and Spin-magnetic field $\mathbf{B}_s$ of C-Sin-EM have the same form as that of Spin motive force $\mathbf{E}_{Qs}$ and Berry curvature $\mathbf{B}_{Qs}$ of Q-Spin-EM.

However, the fundamental differences between the C-Spin-EM and Q-Spin-EM are the definitions of spin-electric field and spin-magnetic field, as well the field equations.

Lagrangian and Hamiltonian

For a non-relativistic non-spinning e-charge Q_e in the EM field, the regular Lagrangian and Hamiltonian are espectively,

$$\mathcal{L}_{reg} = \frac{1}{2}mv^2 + Q_e\mathbf{A}\cdot\mathbf{V} - Q_e\varphi,$$

$$H_{reg} = \frac{1}{2m}\left(p - Q_e\mathbf{A}\right)^2 + Q_e\varphi \tag{3.26}$$

For a rotating mass, the Lagrangian contains its rotation energy, $KE_{rotation} = \frac{1}{2}I\omega^2$

We define the rotation energy of a spinning e-charge as,

$$KE_{spin} \equiv \frac{1}{2}\alpha_1(\mathbf{S}_e)^2 = \frac{1}{2\alpha_1}(\mathbf{L}_s)^2 \tag{3.27}$$

where the spin angular momentum $\mathbf{L}_s \equiv \alpha_1\ \mathbf{S}_e$. Where a_1 is the coefficient.

By the duality between Extended EM and Spin-EM, let's introduce Lagrangian,

$$_{spin} = \frac{1}{2}\alpha_1(\mathbf{S}_e)^2 + a_2 Q_e \mathbf{A}_s \cdot \mathbf{S}_e \tag{3.28}$$

Taking into account the interaction between the velocity and spin-vector-potential, and between the spin and vector potential, we obtain

$$\mathcal{L}_{interaction} = a_3 Q_e \mathbf{A}_s \cdot \mathbf{v} + a_4 Q_e \mathbf{A}\cdot\mathbf{S}_e \tag{3.29}$$

Where a_2 to a_4 are coupling coefficients. Hereafter we ignore those coefficients.

The total Lagrangian of a spinning e-particle in EM fields and Spin-EM fields is

$$\mathcal{L}_{total} = \frac{1}{2}mv^2 + Q_e\mathbf{A}\cdot\mathbf{v} - Q_e\varphi + \frac{1}{2}(\mathbf{S}_e)^2 + Q_e\mathbf{A}_s\cdot\mathbf{S}_e + Q_e\mathbf{A}_s\cdot\mathbf{v} + Q_e\mathbf{A}\cdot\mathbf{S}_e \tag{3.30}$$

In the derivation of Spin-EM, we have replaced the velocity $\mathbf{v}$ by spin $\mathbf{S}_e$ in UMFT. Now we use spin as a "generalized velocity", substituting it into Hamiltonian,

$$H = \Sigma \dot{q}^i \frac{\partial \mathcal{L}_{total}}{\partial \dot{q}^i} - \mathcal{L}_{total} = v\frac{\partial \mathcal{L}_{total}}{\partial v} + (\mathbf{S}_e)\frac{\partial \mathcal{L}_{total}}{\partial \mathbf{S}_e} - \mathcal{L}_{total} \tag{3.31}$$

we obtain the Hamiltonian for spinning e-particles,

$$H = \frac{\left(\mathbf{p} - Q_e\mathbf{A} - Q_e\mathbf{A}_s\right)^2}{2m} + \frac{\left(\mathbf{p}_s - Q_e\mathbf{A}_s - Q_e A\right)^2}{2a_{\mathcal{L}}} + Q_e\varphi + Q_e\varphi_s, \tag{3.32}$$

which describes dynamics of spinning e-particles in both Extended EM and Spin-EM fields. Where the $\mathbf{p}_s$ is a conjugate momentum corresponding to the spin $\mathbf{S}_e$,

$$\mathbf{p}_s = \frac{\partial \mathcal{L}_{total}}{\partial \mathbf{S}_e} \tag{3.33}$$

For the situation, both uniform magnetic field $\mathbf{B}$ and uniform spin-magnetic field $\mathbf{B}_s$ are in z-direction, the vector potential $\mathbf{A}$ and spin-vector-potential $\mathbf{A}_s$ have similar form,

$$\mathbf{A} = \frac{1}{2}\mathbf{B}\times\mathbf{r} \tag{3.34}$$

$$\mathbf{A}_s = \frac{1}{2}\mathbf{B}_s\times\mathbf{r} \tag{3.35}$$

Equation (3.35) shows the relation between the spin-vector potential $\mathbf{A}_s$ and spin-magnetic field $\mathbf{B}_s$ induced by spin, which has the same form as that the vector potential $\mathbf{A}$ induced by magnetic field $\mathbf{B}$, Eq, (3.34).

Spin-Aharonov–Bohm-type Effect and Experiment

The regular Hamiltonian causes the phase shift of Aharonov–Bohm effect. Eq. (3.32) predicts an effect that a spin-vector-potential $\mathbf{A}_s$ induces a phase shift, $\Delta\varphi_{spin}$,

$$\Delta\varphi_s \sim \frac{Q_e}{\hbar}\oint A_s \cdot \mathbf{dr} \tag{3.36}$$

$$\oint A_s \cdot \mathbf{dr} = \iint \mathbf{B}_s \cdot \mathbf{ds} = \iint (\mathbf{S}_s \times \mathbf{E})\cdot\mathbf{ds} = \iint \left\{\mathbf{S}_s \times \left(-\nabla\varphi_e - \frac{\partial \mathbf{A}}{\partial t}\right)\right\}\cdot\mathbf{ds} \tag{3.37}$$

which we denote as the Spin-Aharonov–Bohm-type effect, which is caused by the interaction between e-particles' spin, gradient of electric scalar potential, φ_e, and time changing of magnetic vector potential, $\partial\mathbf{A}/\partial t$.

When a spinning e-particle travelling along the same path P in a region with non-zero $\mathbf{A}$, φ_e and $\partial\mathbf{A}/\partial t$, acquires a total phase shift, $\Delta\varphi_{total}$, which extends Aharonov–Bohm effect, $\Delta\varphi_{AB} = \Delta\varphi_{AB}$,

$$\Delta\varphi_{total} = \Delta\varphi_{AB} + \Delta\varphi_{spin} = \frac{Q_e}{\hbar}\oint \mathbf{A}\cdot\mathbf{dr} + \frac{Q_e}{\hbar}\oint \mathbf{A}_s\cdot\mathbf{dr} \tag{3.38}$$

Remark: Eq. (3.38) shows that the magnitude of a Spin-Aharonov–Bohm-type effect depends on the relative directions between $\mathbf{S}_s$, φ_e, and $\partial\mathbf{A}/\partial t$.

Effects/Predictions of Hamiltonian

Substituting both Equation (3.34) and Equation (3.35) into Equation (3.32), we obtain four different forms,

$$H \approx -\frac{Q_e\mathbf{p}\cdot\mathbf{A}}{m} - \frac{Q_e\mathbf{p}\cdot\mathbf{A}_s}{m} - \frac{Q_e\mathbf{p}_s\cdot\mathbf{A}_s}{a_{\mathcal{L}}} - \frac{Q_e\mathbf{p}_s\cdot A}{a_{\mathcal{L}}} =$$

$$-\frac{Q_e}{2m}\mathbf{B}\cdot\mathbf{L} - \frac{Q_e}{2m}\mathbf{B}_s\cdot\mathbf{L} - \frac{Q_e}{2a_{\mathcal{L}}}\mathbf{B}\cdot\mathbf{L}_s - \frac{Q_e}{2a_{\mathcal{L}}}\mathbf{B}_s\cdot\mathbf{L}_s \tag{3.39a}$$

$$H = -\frac{Q_e}{2m}(\mathbf{B}+\mathbf{B}_s)\cdot\mathbf{L} - \frac{Q_e}{2a_{\mathcal{L}}}(\mathbf{B}+\mathbf{B}_s)\cdot\mathbf{L}_s \quad (3.39b)$$

$$H = -\mathbf{B}\cdot\left(\frac{Q_e}{2m}\mathbf{L}+\frac{Q_e}{2a_{\mathcal{L}}}\mathbf{L}_s\right) - \mathbf{B}_s\cdot\left(\frac{Q_e}{2m}\mathbf{L}+\frac{Q_e}{2a_{\mathcal{L}}}\mathbf{L}_s\right) \quad (3.39c)$$

$$H = -(\mathbf{B}+\mathbf{B}_s)\cdot\left(\frac{Q_e}{2m}\mathbf{L}+\frac{Q_e}{2a_{\mathcal{L}}}\mathbf{L}_s\right) \quad (3.39d)$$

With Hamiltonian, Spin-EM can be converted to its quantum version. The Hamiltonian not only provides classical counterparts/origins of several quantum phenomena, but also predicts several classical effects that may be converted to quantum effects.

With the Hamiltonian, the following effects/phenomena have been predicted.

Zeeman Effect: Angular Momentum L Coupling to Magnetic Field B

The first term on the right-hand side of Equation (3.39a) represents the regular Zeeman effect

$$H_{p-A} \approx -\frac{Q_e}{2m}\mathbf{B}\cdot\mathbf{L} \quad (3.40)$$

Spin-Zeeman Effect (1): Angular Momentum L Coupling to Spin-Magnetic Field B_s

Applying the definition, $\mathbf{B}_s = \mathbf{S}_e \times \mathbf{E}$, the second term of Equation (3.39a) becomes

$$H_{p-A_s} \approx -\frac{Q_e}{m}\mathbf{B}_s\cdot\mathbf{L} = \frac{Q_e}{m}\mathbf{E}\cdot(\mathbf{S}_e\times\mathbf{L}) \quad (3.41)$$

We refer Equation (3.41) as Spin-Zeeman effect (or Extended-Rashba-SOC). Comparing with the regular Rashba-SOC, $H_{Rashba} = \alpha_R\, \mathbf{E}\cdot(\boldsymbol{\sigma}\times\mathbf{p})$).

Remark: The p represents a linear motion; $\mathbf{L} = \mathbf{r}\times\mathbf{p}$ represents an orbiting motion; thus, the term, $\mathbf{S}_e \times \mathbf{L}$, represents indeed a Spin-Orbit-coupling. Actually, when Rashba SOC is applied to several situations, the momentum p is replaced by angular momentum L, which indicates that Rashba SOC is not exactly the Spin-Orbit-coupling.

Spin-Zeeman Effect (2): Spin-Angular Momentum L_s Coupling to Magnetic Field B

The third term of Equation (3.39a) shows the spin-angular momentum $\mathbf{L}_s$ coupling to magnetic field **B**.

$$H_{p_s-A} = -\frac{Q_e}{2a_{\mathcal{L}}}\mathbf{B}\cdot\mathbf{L}_s \quad (3.42)$$

Spin-Zeeman Effect (3): Spin-Angular-Momentum L_s Coupling to Spin-Magnetic Field B_s

The fourth term of Equation (3.39a) shows the Spin-Angular-Momentum $\mathbf{L}_s$, $\mathbf{L}_s \equiv \mathbf{r}\times\mathbf{p}_s$, coupling to Spin-magnetic field $\mathbf{B}_s$.

$$H_{p_s-A_s} = -\frac{Q_e}{2a_{\mathcal{L}}}\mathbf{B}_s\cdot\mathbf{L}_s = -\frac{Q_e}{2a_{\mathcal{L}}}(\mathbf{S}_e\times\mathbf{E})\cdot\mathbf{L}_s = \frac{Q_e}{2a_{\mathcal{L}}}\mathbf{E}\cdot(\mathbf{S}_e\times\mathbf{L}_s) \quad (3.43)$$

Angular Momentum L and Spin-angular-momentum L_s Coupling to Total magnetic Field $(B + B_s)$

Equation (3.39b),

$$H = -\frac{Q_e}{2m}(\mathbf{B}+\mathbf{B}_s)\cdot\mathbf{L} - \frac{Q_e}{2a_{\mathcal{L}}}(\mathbf{B}+\mathbf{B}_s)\cdot\mathbf{L}_s \quad (3.39b)$$

shows that: the angular momentum **L** of a spin-particle couples to both the magnetic **B** field and spin-magnetic field $\mathbf{B}_s$. The spin-angular momentum $\mathbf{L}_s$ of the spin-particle couples to both the magnetic **B** field and spin-magnetic field $\mathbf{B}_s$.

Total Angular Momentum $\left(\frac{Q_e}{2m}\mathbf{L}+\frac{Q_e}{2a_{\mathcal{L}}}\mathbf{L}_s\right)$ Coupling to Magnetic Field B and Spin-Magnetic Field B_s

Equation (3.39c)

$$H = -\mathbf{B}\cdot\left(\frac{Q_e}{2m}\mathbf{L}+\frac{Q_e}{2a_{\mathcal{L}}}\mathbf{L}_s\right) - \mathbf{B}_s\cdot\left(\frac{Q_e}{2m}\mathbf{L}+\frac{Q_e}{2a_{\mathcal{L}}}\mathbf{L}_s\right) \quad (3.39c)$$

shows that: The total-angular momentum $\left(\frac{Q_e}{2m}\mathbf{L}+\frac{Q_e}{2a_{\mathcal{L}}}\mathbf{L}_s\right)$ of an orbiting-spin-particle couples to both the magnetic B field and spin-magnetic field $\mathbf{B}_s$.

Total Angular Momentum $\left(\frac{Q_e}{2m}\mathbf{L}+\frac{Q_e}{2a_{\mathcal{L}}}\mathbf{L}_s\right)$ Coupling to Total magnetic Field $(B + B_s)$

Equation (3.39d) becomes

$$H = -(\mathbf{B}+\mathbf{B}_s)\cdot\left(\frac{Q_e}{2m}\mathbf{L}+\frac{Q_e}{2a_{\mathcal{L}}}\mathbf{L}_s\right) \quad (3.39d)$$

shows that: The total-angular momentum $\left(\frac{Q_e}{2m}\mathbf{L}+\frac{Q_e}{2a_{\mathcal{L}}}\mathbf{L}_s\right)$ of an orbiting-spin-particle couples to total of magnetic B field and spin-magnetic field $\mathbf{B}_s$, $(\mathbf{B} + \mathbf{B}_s)$.

Effects of Lorentz Force on Spin

We have predicted the effects of Lorentz Force on Spin:

1. Classical Origin of Aharonov–Casher effect
2. Spin-Stark Effect
3. Magnetic-Rashba-type SOC.

Spin-Lorentz-type Force and Effects

Spin-Lorentz-type Force

Based on the type-2 duality between Extended EM and Spin-EM, we postulate that, beside the dualities between fields and field equations, there is a type-2 duality between Lorentz force and a spin related force, denoted as spin-Lorentz-type force, i.e., under the transformation, $E\leftrightarrow E_s$, $B\leftrightarrow B_s$, $v \leftrightarrow S_e$, Lorentz force, $F_{Lorentz}$ (abbreviate F_L), converts to Spin-Lorentz-type force, $F_{spin\text{-}Lorentz}$ (abbreviate F_{SL}), and vice versa,

$$\mathbf{F}_{SL} = m\frac{dv}{dt} = Q_e\mathbf{E}_s + Q_e\mathbf{v}\times\mathbf{B}_s + Q_e\mathbf{S}_e\times\mathbf{B} + Q_e\mathbf{S}_e\times\mathbf{B}_s \quad (3.45)$$

We refer "$Q_e\, \mathbf{E}_s$" as spin-electric force, "$Q_e\, \mathbf{v}\times\mathbf{B}_s$", "$Q_e\, \mathbf{S}_e \times \mathbf{B}$", and "$Q_e\, \mathbf{S}_e\times\mathbf{B}_s$" as spin-magnetic force.

Effects of Spin-Lorentz-type Force

1. **Magnetic Spin-Aharonov-Casher-type Effect**

Taking the cross and dot products of Spin-Lorentz-type force and spin of a test e-particle, respectively, we obtain

$$\mathbf{S}_s\times\mathbf{F}_{SL} = Q_e\mathbf{S}_s\times\mathbf{E}_s + Q_e\mathbf{S}_s\times(\mathbf{v}\times\mathbf{B}_s) + Q_e\mathbf{S}_s\times(\mathbf{S}_e\times\mathbf{B}) + Q_e\mathbf{S}_s\times(\mathbf{S}_e\times\mathbf{B}_s), \quad (3.46)$$

$$\mathbf{S}_s\cdot\mathbf{F}_{SL} = Q_e\mathbf{S}_s\cdot\mathbf{E}_s + Q_e\mathbf{v}\times\mathbf{B}_s + Q_e\mathbf{S}_s\cdot(\mathbf{S}_e\times\mathbf{B}) + Q_e\mathbf{S}_s\cdot(\mathbf{S}_e\times\mathbf{B}_s) \quad (3.47)$$

When converting to quantum, the first term of Eq. (3.46) implies that spin-electric field $\mathbf{E}_s$ shifts the phase of a spinning e-particle.

Remark: (1) Denoted Equation (3.46) as the Aharonov-Casher-type effect. (2) Equation (3.47) is an Extended Spin-Orbit-Electric field Coupling.

2. **Effect of Spin-Lorentz-type force**

The effects of Spin-Lorentz-type force include:

$$\mathbf{v}\times\mathbf{F}_{SL} = Q_e\mathbf{v}\times\mathbf{E}_s + Q_e\mathbf{v}\times(\mathbf{v}\times\mathbf{B}_s) + Q_e\mathbf{v}\times(\mathbf{S}_e\times\mathbf{B}) + Q_e\mathbf{v}\times(\mathbf{S}_e\times\mathbf{B}_s). \quad (3.48)$$

$$\mathbf{v}\cdot\mathbf{F}_{SL} = Q_e\mathbf{v}\cdot\mathbf{E}_s + Q_e\mathbf{v}\cdot(\mathbf{v}\times\mathbf{B}_s) + Q_e\mathbf{v}\cdot(\mathbf{S}_e\times\mathbf{B}) + Q_e\mathbf{v}\cdot(\mathbf{S}_e\times\mathbf{B}_s) \quad (3.49)$$

$$\mathbf{r}\times\mathbf{F}_{SL} = Q_e\mathbf{r}\times\mathbf{E}_s + Q_e\mathbf{r}\times(\mathbf{v}\times\mathbf{B}_s) + Q_e\mathbf{r}\times(\mathbf{S}_e\times\mathbf{B}) + Q_e\mathbf{r}\times(\mathbf{S}_e\times\mathbf{B}_s). \quad (3.50)$$

$$\mathbf{r}\cdot\mathbf{F}_{\mathbf{SL}} = Q_e\mathbf{r}\cdot\mathbf{E}_s + Q_e\mathbf{r}\cdot(\mathbf{v}\times\mathbf{B}_s) + Q_e\mathbf{r}\cdot(\mathbf{S}_e\times\mathbf{B}) + Q_e\mathbf{r}\cdot(\mathbf{S}_e\times\mathbf{B}_s) \quad (3.51)$$

3. **More effects of Spin-Lorentz-type Force**

The following effects have been studied.

- Dual-Hall Effect/Topological Insulator;
- Spin-Extended-Hall Effect/Topological Insulator
- Spin-Extended-Hall Effect Having Zero Longitudinal Resistivity
- Spin-Extended-Hall effect Contributing to GMR/TMR Effect,
- Spin-Extended-Hall Effect Contributing to Spin Hall Effect
- Spin-Extended-Hall effect Contributing to Anomalous-Hall Effect
- Spin-Temperature Dependence of Extended-Hall Effect;
- Spin-Potential-Coupling Contributing to Aharonov–Bohm Effect.

Spin-Lagrangian-Lorentz-type Force and Effects

Spin-Lagrangian-Lorentz-type Force

Starting with Spin-Lagrangian's equation $\frac{d}{dt}\frac{\partial\mathcal{L}_{total}}{\partial v} = \frac{\partial\mathcal{L}_{total}}{\partial r}$ where $\mathbf{L}_{total}$ is,

$$\mathcal{L}_{total} = \frac{1}{2}mv^2 + Q_e\mathbf{A}\cdot\mathbf{v} - Q_e\varphi + \frac{1}{2}a_{\mathcal{L}}(\mathbf{S}_e)^2 + Q_e\mathbf{A}_s\cdot\mathbf{S}_e - Q_e\varphi_s + Q_e\mathbf{A}\cdot\mathbf{S}_e + Q_e\mathbf{A}_s\cdot\mathbf{v} \quad (3.52)$$

The total Lagrangian-Lorentz-type Force F_TLL

$$\frac{1}{Q_e}\mathbf{F}_{TLL} = \frac{1}{Q_e}(\mathbf{F}_L + \mathbf{F}_{sL}) = \mathbf{E} + \mathbf{v}\times\mathbf{B} + a\mathbf{E}_s + a\mathbf{v}\times\mathbf{B}_s + a\mathbf{S}_e\times\mathbf{B}_s + a\mathbf{S}_e\times\mathbf{B} \quad (3.53)$$

The "a" is a coefficient.

Effects of Spin-Lagrangian-Lorentz-type Force

We predict the effects of Spin-Lagrangian-Lorentz-type Force

Spin-Potential-Coupling Force Contributing to Aharonov–Bohm Effect

Let's consider a beam of spinning e-particles shooting at a solenoid. The spin e-particles go around the solenoid and experience the spin-potential-coupling force and thus, split into opposite direction, due to the orientations of spins. The spin-potential-coupling force contributes to Aharonov–Bohm Effect (Figure 3.1).

Remark: The Spin-Lagrangian-Lorentz-type force, $\frac{1}{Q_e}\mathbf{F}_{LL} \sim (\mathbf{S}_s\cdot\nabla)\mathbf{A}$, contributes to the A-B effect, which implies that force is a partial cause of the A-B effect of Hamiltonian.

Spin-Larmor-type Precession

Lagrangian-Lorentz-type force $\mathbf{F}_{LL} = Q_e\mathbf{S}_s\times(\mathbf{S}_s\times\mathbf{E})$ induces Spin-Larmor-type precession,

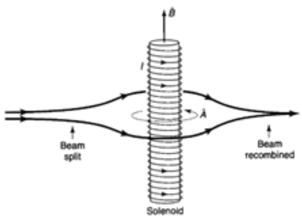

Figure. 3.1. *Setup: Spin-Potential-Coupling Force Contributing to Aharonov–Bohm Effect*

$$\frac{d\mathbf{S}_e}{dt} = \Omega\times\mathbf{S}_s, \quad (3.54)$$

$$\Omega \equiv Q_e(\mathbf{S}_s\times\mathbf{E}) \quad (3.55)$$

Spin-magnetic force is the mechanism of Rashba Effect;

The work done by Spin-magnetic field $\mathbf{B}_s$ is,

$$\mathbf{r}\cdot\mathbf{F}_{LL} = Q_e\mathbf{r}\cdot(\mathbf{v}\times\mathbf{B}_s) + Q_e\mathbf{r}\cdot(\mathbf{S}_s\times\mathbf{B}_s) = \frac{Q_e}{m}\mathbf{E}\cdot[\mathbf{L}\times\mathbf{S}_s] + \frac{Q_e}{m}\mathbf{E}\cdot[\ell_s\times\mathbf{S}_s] \quad (3.56)$$

where $\ell_s \equiv m\mathbf{r}\times\mathbf{S}_s$ The first term is the Extended Rashba SOC.

The underlying mechanism of the Rashba effect is the spin-magnetic force.

Spin-magnetic-Rashba-type SOC

The Power of Lagrangian-Lorentz-type force acting on spin is,

$$\mathbf{v}\cdot\mathbf{F}_{LL} = Q_e\mathbf{v}\cdot(\mathbf{S}_s\times\mathbf{B}_s) = \frac{Q_e}{m}\mathbf{B}_s\cdot(\mathbf{p}\times\mathbf{S}_s) \quad (3.57)$$

We refer the term $\frac{Q_e}{m_e}\mathbf{B}_s\cdot(\mathbf{S}_s\times\mathbf{p})$ as the Spin-magnetic-Rashbe-type SOC.

Classical Origin of Aharonov–Casher effect

We suggest that Lorentz force acts on spin as,

$$\mathbf{S}_s\times\mathbf{F}_L = Q_e\mathbf{S}_s\times\mathbf{E} + \frac{Q_e}{m_e}\mathbf{S}_s\times(\mathbf{p}\times\mathbf{B}) \quad (3.58)$$

The first term provides a classical origin of the Aharonov–Casher effect.

Substituting the identity, **A × (B × C) = (A · C)B - (A · B) C**, into Equation, we obtain

$$\mathbf{S}_s\times\mathbf{F}_L = Q_e\mathbf{S}_s\times\mathbf{E} + \frac{Q_e}{m_e}(\mathbf{S}_s\cdot\mathbf{B})\mathbf{p} - \frac{Q_e}{m_e}(\mathbf{S}_s\cdot\mathbf{p})\mathbf{B} \quad (3.59)$$

Substituting the identity, **A × (B × C) = (A × B) × C + B × (A × C)**, into Eq. (3.54), we obtain,

$$\mathbf{S}_s\times\mathbf{F}_L = Q_e\mathbf{S}_s\times\mathbf{E} + \frac{Q_e}{m_e}(\mathbf{S}_s\times\mathbf{p})\times\mathbf{B} + \frac{Q_e}{m_e}\mathbf{p}\times(\mathbf{S}_s\times\mathbf{B}) \quad (3.60)$$

Substituting the definition of the E_s field into Eq. (3.55), we obtain,

$$\mathbf{S}_s\times\mathbf{F}_L = Q_e\mathbf{S}_s\times\mathbf{E} + \frac{Q_e}{m_e}(\mathbf{S}_s\times\mathbf{p})\times\mathbf{B} - \frac{Q_e}{m_e}\mathbf{p}\times\mathbf{E}_s \quad (3.61)$$

Spin-Stark Effect; Magnetic-Rashba-type SOC

We can find how Lorentz force affects spin differently,

$$\mathbf{S}_s\cdot\mathbf{F}_L = Q_e\mathbf{S}_s\cdot\mathbf{E} + \frac{Q_e}{m_e}\mathbf{S}_s\cdot(\mathbf{p}\times\mathbf{B}) = Q_e\mathbf{S}_s\cdot\mathbf{E} + \frac{Q_e}{m_e}\mathbf{B}\cdot(\mathbf{S}_s\times\mathbf{p}) \quad (3.62)$$

The $Q_e\, S_s{\cdot}E$ causes spin-Stark effect. The $\frac{Q_e}{m_e}\mathbf{B}\cdot(\mathbf{S}_s\times\mathbf{p})$ is a magnetic-Rashba-type SOC.

Summery and discussion

Combining UMFT, Coulomb's law and spin, we derived Spin-EM in the perspective of fundamental physics.

1. Spin-EM is powerful and fruitful, and achieves the following:
2. Derives spin wave.
3. Derives Spin-Lorentz-type force and Lagrangian-Lorentz-type force, which, for 3D model, cause Dual-Hall Effect, Extended-Hall effect, Lagrangian-Hall-type Effect, and Temperature Dependence of Extended-Hall

effect; also cause Extended Rashba SOC.

4. Extended-Hall effect contributes universally to zero longitudinal Hall coefficient/resistivity, GMR/TMR, Anomalous Hall effect, Spin Hall effect, and topological insulator.
5. Predicts Spin-Potential-Coupling-Induces force that contributes to Aharonov–Bohm Effect.
6. Provides counterparts of Aharonov-Bohm effect; Aharonov-Casher effect; Stark effec; Larmor Precession.

Proposes several experiments to test proposed effects, such as, definition of spin-electric and spin-magnetic fields, Spin-Aharonov–Bohm effect, Dual-Hall Effect/Topological Insulator, whether ρ_(ext-xx) (B)≈0 of GMR/TMR, Spin-Potential-Coupling-Induced force, Zeeman effect/Extended-Rashba SOC.

We argue that the powerfulness and fruitfulness are evidences supporting Spin-EM. The Spin-EM predicts experimental results, which need to be tested.

The mathematical identities lead to the physical dualities. UMFT provides mathematical origins of physical dualities between Maxwell-EM, Gravito-EM and Spin-EM. We suggest that those physical forces are the physical dual to each other. UMFT provides mathematical origins of physical dualities between Maxwell EM, Gravito-EM, and Spin-EM. They are all derived from UMFT and thus, they have the same symmetry, the U(1) symmetry, and can be unified in the frame of UMFT.

References

1. Hui Peng, Gravity: WEP, Gauge Theory, Quantization, Unification, Intern J. of Phys. 8(3), 90-104. DOI: 10.12691/ijp-8-3-2 (2020).
2. Hui Peng, "Maxwell Equations Derived from Coulomb' Law vs. Maxwell-type Gravity Derived fromNewton's Law." International Journal of Physics, vol. 9, no. 1: 1-27. doi: 10.12691/ijp-9-1-1 (2021).

Appendix: Spin-Gravity

We have proposed Spin-EM in the Section 3. In this Section we show that the spin of a gravitational mass/charge (g-charge) produces Spin-gravity fields.

A.1. Definitions of Spin-gravity Fields

We have shown that a mass (g-charge, Q_g), produces a gravito-electric-field g, $\nabla \cdot g = - Q_g$, while a moving g-charge (velocity v) produces a gravito-magnetic-field B_g, $\nabla \times B_g = - Q_g \mathbf{v}$.

Now let us consider a spinning g-charge, for example a neutron, characterized by both g-charge Q_g and spin S_g. In an external gravito-electric-field g and an external gravito-magnetic field B_g, the spin S_g produces the spin-gravito-electric-field g_s and the spin-gravito-magnetic field B_{gs}:

$$\mathbf{B_{gs} \equiv S_g \times g}, \quad \text{(A.1)}$$

$$g_s \equiv - S_g \times B_g. \quad \text{(A.2)}$$

The definitions, Equation (A.1) and Equation (A.2), state that: by interacting with a gravitation field g and a gravito-magnetic field B_g respectively, the spin of a g-charge produces a spin-gravito-magnetic field B_{gs} and a spin-gravitation field g_s, respectively. Subscript "s" indicates that the quantity is related to spin. The B_{gs} and g_s are type-2 dual to the B_g and g fields, respectively. The B_{gs} and g_s are "Second order axial vector field" (SAF) and "Third order axial vector field" (TAF) respectively.

A.2. Spin-gravity

Assuming that the spin S_g of a g-charge is constant. Let the "T" is the external gravito-electric-field g and the external gravito-magnetic field B_g respectively. Equation (2.5) to Equation (2.7) give respectively,

$$\nabla \times (S_g \times g) = \nabla \times B_{gs} = S_g (\nabla \cdot g) - (S_g \cdot \nabla) g, \quad \text{(A.3a)}$$

$$\nabla \times (B_{gs}) = S_g (\nabla \cdot g) + ((\partial g_s)/\partial t) + [\nabla (S_g \cdot g) - 2(S_g \cdot \nabla) g - S_g \times (\nabla \cdot g)]. \quad \text{(A.3b)}$$

$$\nabla \times (S_g \times B_g) = - \nabla \times g_s = S_g (\nabla \cdot B_g) - (S_g \cdot \nabla) B_g, \quad \text{(A.4a)}$$

$$\nabla \times (g_s) = -S_g (\nabla \cdot B_g) - (\partial B_{gs})/\partial t - [\nabla (S_g \cdot B_g) - 2(S_g \cdot \nabla) B_g - S_g \times (\nabla \cdot B_g)]. \quad \text{(A.4b)}$$

$$\nabla \cdot (S_g \times g) = \nabla \cdot B_{gs} = - S_g \cdot (\nabla \times g). \quad \text{(A.5a)}$$

$$\nabla \cdot (S_g \times B_g) = - \nabla \cdot g_s = - S_g \cdot (\nabla \times B_{gs}). \quad \text{(A.5b)}$$

Spin-gravity in this form has the same form as that of Spin-EM.

A.3. Spin-Poynting-Type-Theorem of Spin-Gravity

Equation (2.2), $\nabla \cdot (R \times T) = (\nabla \times R) \cdot T - R \cdot (\nabla \times T)$, shows the Poynting-type theorem of Spin-gravity:

(*0) Let R=g and T=B_g, Equation (2.2) gives the normal Poynting theorem:

$$\nabla \cdot (g \times B_g) = (\nabla \times g) \cdot B_g - g \cdot (\nabla \times B_g). \quad \text{(A.6a)}$$

(*1) Let R=g_s and T=B_gs, Equation (2.2) gives the spin-Poynting theorem (a):

$$\nabla \cdot (g_s \times B_{gs}) = (\nabla \times g_s) \cdot B_{gs} - g_s \cdot (\nabla \times B_{gs}) \quad \text{(A.6b)}$$

(*2) Let R=g_s and T=B, Equation (2.2) gives the spin-Poynting theorem (b)

$$\nabla \cdot (g_s \times B) = (\nabla \times g_s) \cdot B - g_s \cdot (\nabla \times B) \quad \text{(A.6c)}$$

(*3) Let R=g and T=B_gs, Equation (2.2) gives the spin-Poynting theorem (c):

$$\nabla \cdot (g \times B_{gs}) = (\nabla \times g) \cdot B_{gs} - g \cdot (\nabla \times B_{gs}) \quad \text{(A.6d)}$$

Note: the spin S_e generates the spin-magnetic field, which is conceptually different from that of Rashba-induced-Spin-EM.

A.4. Spin-Lorentz-type gravitational force

By analog to the Lorentz force, we propose the Spin-Lorentz-type gravitational force on a moving spin-g-charge q_g:

$$F = (q_g\, g + q_g\, v \times B_g) + \nabla (q_g\, g_s + q_g\, v \times B_{gs} + q_g\, S_g \times B_g + q_g\, S_g \times B_{gs}). \quad \text{(A.7)}$$