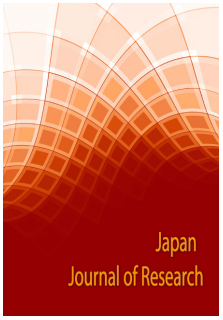
Japan Journal of Research

Review Article




# An Evaluation to The Taylor Critical Strip Finding Method Against the Analytic Laurent Series One

Lam(Carson) Kai Shun

British National Oversea, B.Sc., M.Sc.(ComEngine), Faculty of Engineering, M.Sc. (M.I.T.E.), Faculty of Education, Former Higher Doctorate -- Doctor of Science Finalist, University of Hong Kong, Fellow, Dig Fellow, Distinguished Fellow, India Journals.

Correspondence

**Lam (Carson) Kai Shun,**

British National Oversea, B.Sc., M.Sc. (ComEngine), Faculty of Engineering, M.Sc. (M.I.T.E.), Faculty of Education, Former Higher Doctorate -- Doctor of Science Finalist, University of Hong Kong,

- Received Date: 10 May 2026
- Accepted Date: 04 July 2026
- Publication Date: 13 July 2026



## Abstract

*In the present paper, this author will try to revisit the proof of my Riemann Hypothesis by extending the Taylor Series Approximation to the Laurent Series which is in fact can overcome the deficiencies in the divergence problem for the classical Riemann Zeta Function for the case of $0 < s < 1$. In addition, this author also introduces the multiplication factor of (1-2(1-s)) to the zeta function when combined with each other to form the Dirichlet Eta function and hence we may reuse nearly all of my previous papers' computed results.*

*Next this author also tries to optimize the Riemann Zeta Function's empirical root model equation and compare with the inverse power index 2 and 3. It is shown that if we multiply a factor to the inner part of the cotangent function, then both of the empirical equations can be minimized while there may other parts for the maximization which may indeed constitute the business simplex method "primal & dual" problems.*

*The last issue of this paper is to focus in the number theory, elliptical curves and elliptical functions together with the cryptography. In a simplified and less abstract language, there is a one-to-one correspondence between the elliptical curves and the elliptical functions. Hence, in one way, we may solve the Weierstrass P function and establish the corresponding Fourier series with a suitable period and lattice such that we may get the essential congruence modular formula for the cryptography that has been shown in the Apostol and Washington's books.*

*On the other hand, from the respective elliptical curves, we may also obtain an algorithmic procedures (that will be shown in the section before conclusion) for solving the integral modular on the elliptical curves and thus establish the cryptography.*

*In brief, the present paper will extend the proof for the Riemann Hypothesis for the case of $0 < s < 1$. The paper also optimizes the empirical Riemann Zeta Root model equations. It also helps us to get an application in the field of elliptical curve cryptography from the proof of truth to the Riemann Hypothesis.*

## Introduction

Since my first publication in the series about the Riemann Hypothesis, there are lots of discussion about my trials to the proof of the such hypothesis. In fact, the controversy lays in the fact that whether we – mathematicians should allow the application of Taylor Series approximation to the Riemann Zeta Function. Actually, in the present paper, this author will extend the Taylor Series approximation to the Laurent Series for the complex valued function with the negative power. Moreover, this author will also amend the proof of the power index in the Riemann Zeta Function for the case of $0 < s < 1$ with the introduction of the multiple (1-2(1-s)). Hence, it is hope that this author's proof to Riemann Hypothesis will be more fully and completed for those interested readers when the present paper is compared with my series of previous papers.

## Mathematical Computational Details

By considering up to the quadratic order of the Taylor Expansion for the Dirichlet Era function $\sum_{n=1}^{\infty}\frac{-1^{(n-1)}}{n^s}$, we get

$$h=\frac{e^{(a-1)\pi I}}{e^{(u+vi)ln(a)}}+\frac{\left(e^{(a-1)\pi I}\pi I-\frac{e^{(a-1)\pi I}(u+vi)}{a}\right)(x-a)}{e^{(u+vi)ln(a)}}+\frac{\left(\begin{matrix}\frac{-e^{(a-1)\pi I}\pi^2}{2}-\frac{e^{(a-1)\pi I}\left(u^2+2uvi+vi^2-u-vi\right)}{2a^2}\\-\frac{e^{(a-1)\pi I}(\pi aI-u-vi)(u+vi)}{a^2}\end{matrix}\right)(x-a)^2}{e^{(u+vi)ln(a)}}$$

**Citation**: Lam KS. An Evaluation to The Taylor Critical Strip Finding Method Against the Analytic Laurent Series One. Japan J Res. 2026;7(5):192.

With using the solve command for the Maple Soft, we have the following root model equations:

$$\left\{a=a, u=\frac{I\pi a^2 - I\pi a x - avi + vix - \frac{3a}{2} + \frac{x}{2} + \frac{\sqrt{-4I\pi a^3 + 8I\pi a^2 x - 4I\pi a x^2 + a^2 - 6xa + x^2}}{2}}{-x+a}, vi=vi, x=x\right\}$$

$$\left\{a=a, u=\frac{I\pi a^2 - I\pi a x - avi + vix - \frac{3a}{2} + \frac{x}{2} + \frac{\sqrt{-4I\pi a^3 + 8I\pi a^2 x - 4I\pi a x^2 + a^2 - 6xa + x^2}}{2}}{-x+a}, vi=vi, x=x\right\}$$

where the real part of the modeled u is: $\frac{(3*a+x)}{2*(-x+a)}$. In fact, for $u = 1/2$ or $u = 3/2$ when either $x \to \infty$ or $a \to \infty$ respectively. This author notes that the square root part of the above two paired model answers is in fact a complex value after simplification, i.e. $\frac{1}{2}\sqrt{1-4aI\pi}$. Hence, this author concludes that the model answer for the real part of the Dirichlet Eta function is either $u = 1/2$ or u = 3/2. In addition, the Dirichlet Eta function is well defined for all complex numbers s = u + vI even for 0 < u < 1. The Eta function is only undefined for s = 1. Furthermore, the roots of the Eta function is just the same as the Zeta function as their equations are only different by an alternative $(-1)^{n-1}$ or just $e^{i(n-1)\pi}$ although their root model equation may be quite different. However, the advantage of introducing the Dirichlet Eta function is: The Eta function is well defined even for 0 < s < 1 except x = 1. Hence, the Eta function is convenient for those interested to investigate the rooting properties of the true Riemann Zeta function.

## Use the Laurent Series to find the Riemann Zeta Non-Trivial Zeros's Root Model Equation:

With the application of the Laurent series to the Maple Soft 2024 Home Edition to the Riemann Zeta Function $\sum_{i=1}^{\infty} 1/i^s$, where $s = u + vI$ with u, v ∈ ℝ,

$$laurent\left(\frac{1}{x^{(u+v*I)}}, x=a\right)$$

we may get the following result:

$$\frac{1}{e^{((u+v*I)*ln(a))}} - \frac{(u+v*I)*(x-a)}{\left(a*e^{((u+v*I)*ln(a))}\right)} + \frac{\left(\frac{-\left(-v^2+2*I*u*v-v*I+u^2-u\right)}{\left(2*a^2\right)} + \frac{(u+v*I)^2}{a^2}\right)*(x-a)^2}{e^{((u+v*I)*ln(a))}}$$

$$+ \frac{\left(\frac{-\left(-3*u*v^2+3*v^2+3*I*u^2*v-v^3*I-6*I*u*v+u^3+(2*v)*I-3*u^2+2*u\right)}{\left(6*a^3\right)} + \frac{\left(\left(-v^2+2*I*u*v-v*I+u^2-u\right)*(u+v*I)\right)}{\left(2*a^3\right)} - \frac{\left(\left(2*I*u*v+v*I+u^2-v^2+u\right)*(u+v*I)\right)}{\left(2*a^3\right)}*(x-a)^3\right)}{exp}((u+v*I)*ln(a))+...+O\left((x-a)^6\right)$$

By employing the "solve" function from the Maple Soft, we may have:

$$solve\left(\frac{1}{e^{((u+v*I)*ln(a))}} - \frac{(u+v*I)*(x-a)}{\left(a*e^{((u+v*I)*ln(a))}\right)} + \frac{\left(\frac{-\left(-v^2+2*I*u*v-v*I+u^2-u\right)}{\left(2*a^2\right)} + \frac{(u+v*I)^2}{a^2}\right)*(x-a)^2}{exp}((u+v*I)*ln(a)), u\right)$$

we obtain the following root model equations:

$$\frac{\left(-a*v*I+v*x*I-\frac{(3*a)}{2}+\frac{x}{2}+\sqrt{\left(a^2-6*a*x+x^2\right)}/2\right)}{(-x+a)}, \frac{\left(-a*v*I+v*x*I-\frac{(3*a)}{2}+\frac{x}{2}-\frac{\sqrt{\left(a^2-6*a*x+x^2\right)}}{2}\right)}{(-x+a)}$$

In fact, the real part of the above solved model answer is:

$$\frac{(3*a)}{2*(-x+a)} + \frac{x}{2*(-x+a)} + \frac{\sqrt{\left(a^2-6*a*x+x^2\right)}}{2*(-x+a)}$$

or

$$\frac{(3*a)}{2*(-x+a)} + \frac{x}{2*(-x+a)} - \frac{\sqrt{\left(a^2-6*a*x+x^2\right)}}{2*(-x+a)}$$

After simplifying, we have:

$$\frac{(3*a+x)}{2*(-x+a)} \pm \frac{1}{2}\sqrt{1-\frac{4ax}{(a-x)^2}}$$

By taking limit for a → ∞ together with x → ∞ respectively, we may get the answer for the real part u in the root of the Riemann Zeta non-trivial zeros' model:

3/2 ±1/2 or 1/2 ±1/2 (when supplemented with the real part of the above Dirichlet Eta function's root model).

Obviously, the above root model equations tell us that the real part of the non trivial zeta critical lines are just those lines centered at x = 1/2 or x = 3/2 with the width of 1/2.

In reality, the above results are just the same as this author's previous finding in my previous papers through applying the Taylor series and I will NOT repeat. (But in this author's previous papers have missed to consider such geometric width of ±1/2).

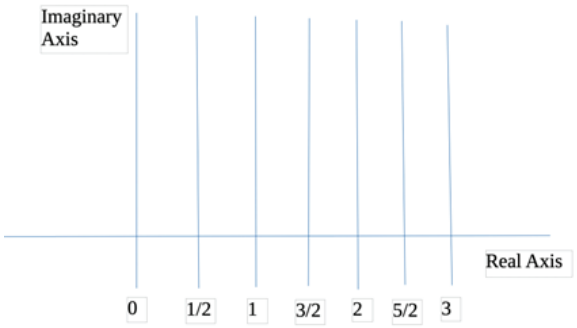



***Figure 1:*** *A Modeled Geographical Distribution Map of the feasible Riemann non-trivial zeta zeros, the point x = 1 is the singularity; x = 1/2 is the critical line with all non-trivial zeta zeros with x = ½ ± ½ as the width. Similar case dose apply for 3/2 (±1/2) which is the false critical line (lies in the zero free zone) obtained by solving both of the Laurent series of the Zeta function from the Dirichlet Eta function. 5/2 (± 1/2) is also the false critical line lies in the zero free zone obtained by solving the parametric equation of the complex number {s = u+vI | u = 1-t where 0 < t < 1} for the classical Riemann Zeta function ξ(s) through the Laurent series of the Zeta function; 2 & 3 both lie in the zero free region and are also the false critical line while ξ(2) = π2/6.*

## A New and Amended Proof for the Case of {z = u + vI | u ∈ ℝ/{0.5} where 0 < u = (1-t) < 1 and v ∈ ℝ about the prime gap

First, let us multiply the following well-known formulas for the Riemann Zeta function by $1\text{-}2^{1\text{-}s}$:

$$\left(1-2^{1-s}\right)\prod_{i=1}^{\infty}\left(z-z_i\right)^s = \left(1-2^{1-s}\right)\xi(s) = \eta(s) = \left(1-2^{1-s}\right)\sum_{n=1}^{\infty}1/n^s$$

$$= \left(1-2^{1-s}\right)\prod_{j=1}^{\infty}\left(1-1/\,prime_j^{\,s}\right)^{-1}$$

for 0 < s < 1.

Actually, for 0 < s <1, according to the Apostol, Introduction to Analytic Number Theory, the defi-nition should be,

$$\xi(s) = \lim_{x\to\infty}\left(\sum_{n\le x}\frac{1}{n^s} - \frac{x^{1-s}}{1-s}\right)$$

In practice, if we subtract 21-s and then multiply the values of every 2n term (for n = 1, … ∞) in the series $\sum_{n=1}^{\infty}1/n^s$, then the resultant series $\sum_{i=1}^{j}\frac{(-1)^{(i-1)}}{i^s}$ converges to $(1 - 2^{1\text{-}s})\ \xi\ (s) = \eta(s)$.

(N.B. NOT all of the divergence(s) to infinity imply the series $-\sum_{n=1}^{\infty}1/n$ which is the only one of infinite many cases of divergent series.)

That is, although $\sum_{n=1}^{\infty}1/n^s$ diverges for 0 < s < 1 and s = 1, we may still turn a way round by considering the Dirichlet Eta equation, which is:

$$\sum_{i=1}^{j}\frac{(-1)^{(i-1)}}{i^s} = \sum_{i=1}^{j}\frac{e^{(i-1)I\pi}}{i^s} = \left(1-2^{1-s}\right)\xi(s) = \eta(s) \text{ for } 0 < s < 1 \text{ and } s \neq 1,$$

or we may reuse our proofs in my previous published papers like:

$$\sum_{i=1}^{j}1/i^s = r*\cot[j - \tan\text{-}1(v/u] = r*\{[(1-t)*\cot(j) + v] / [u - v*\cot(x)]\}$$

$$\sum_{n=1}^{j}\frac{-1^{(n-1)}}{n^{s}} = r*cot[j\ -\ tan^{-1}(2k\pi-)]\ or\ r*cot[j\ -\ tan^{-1}((2k-1)\pi-\frac{v}{u})]$$

$$r*\frac{ucot(j)+(2k\pi-v)}{u-[(2k\pi-v)cot(j)]}\ \text{or}\ r*\frac{ucot(j)+((2k-1)\pi-v)}{u-[((2k-1)\pi-v)cot(j)]}$$

Then suppose u = 1-t where 0 < t < 1, we may have:

$$\sum_{n=1}^{j}\frac{-1^{(n-1)}}{n^{s}} = r*\frac{(1-t)cot(j)+(2k\pi-v)}{(1-t)-[(2k\pi-v)cot(j)]}\ \text{or}\ r*\frac{(1-t)cot(j)+((2k-1)\pi-v)}{(1-t)-[((2k-1)\pi-v)cot(j)]}$$

where r = |s| = (|u|² + |v|²)1/2 and u = |u| or replaced i by k as the difference between the harmonic series and the $\sum_{i=1}^{j}1/i^{s}$ is just by a rotation plus a zoom in or out.

$$P*_{j+1} = \frac{\left[(1-2^{1-s})\prod_{i=1}^{j}(z-z_i)^{s}*(z\ -\ z_{j+1})^{s}\right]}{\left[(1-2^{1-s})\sum_{i=1}^{j+1}1/i^{s}\right]}$$

$$=\prod_{i=1}^{j}(z-z_i)^{s}*\ (z\ -\ z_{j+1})^{s}\ /r*\frac{ucot(j)+(2k\pi-v)}{u-[(2k\pi-v)cot(j)]}$$

$$=[u\ -\ v*cot(j+1)]*\left[\prod_{i=1}^{j}(z-z_i)^{s}*\ (z\ -\ z_{j+1})^{s}\right]/\{r*[(1-t)*cot\ (j+1)\ +\ v]\}$$

i.e. If the s takes the value: s = (1-t) + v*i, then there is a factor of

[(1-t) – v*cot(j+1)] / {r *[(1-t)*cot(j+1)+v]}.

$P*_{j+1}$ = [(1-t) – v*cot(j+1)] / {r *[(1-t)*cot(j+1)+v]} * $P_{j+1}$

Hence by rearrange those terms, we have:

$P*_{j+1}$ = [$P_{j+1}$]*([(1-t) – v*cot(j+1)] / {r *[(1-t)*cot(j+1)+v]})

i.e. the difference between s = 1 or $P_{j+1}$, and s = (1-t) + v*I or $P*_{j+1}$ is only by the factor of ([(1-t) – v*cot(j+1)] / {r *[(1-t)*cot(j+1)+v]}). Or to transform the $P_{j+1}$ into $P*_{j+1}$, we have to multiple the factor ([(1-t) – v*cot(j+1)] / {r *[(1-t)*cot(j+1)+v]}).

Let W = ([(1-t) – v*cot(j+1)] / {r *[(1-t)*cot(j+1)+v]}) ------- (*”*)

Next, we may have:

(1-$2^{1-(1-t)}$) ξ(1-t) = (1-$2^{1-(1-t)}$) / [ (1- (1/2)$^{(1-t)}$) ( 1- (1/3)$^{(1-t)}$) … ( 1- (1/Prime $_{j}$)$^{(1-t)}$)][1- (1/Prime $_{j+1}$)$^{(1-t)}$]

(1-$2^{1-(1-t)}$)[1- (1/Prime $_{j+1}$)$^{(1-t)}$] = (1-$2^{1-(1-t)}$) / [ξ(1-t))*[ (1- (1/2)$^{(1-t)}$)( 1- (1/3)$^{(1-t)}$)…

( 1- (1/Prime $_{j}$)$^{(1-t)}$]

But by the assumption,

$$\left(1-2^{1-(1-t)}\right)\sum_{i=1}^{j}1/i^{(1-t)} = \left(1-2^{1-(1-t)}\right)[\ \left(1-\ (1/2)^{(1-t)}\right)\left(\ 1-\ (1/3)^{(1-t)}\right)...\left(1-\ (1/Prime_j)^{(1-t)}\right]\ =\ P_j*,$$

hence

[1- (1/Prime $_{j+1}$)$^{(1-t)}$] = 1 / [$P_j$*]²,

Rearrange gives, Prime $_{j+1}$ = 1 / (1 – 1/[$P_j$*]²)$^{1/(1-t)}$ = [ ($P_j$*)2 / ( $P_j$*)² - 1]$^{1/(1-t)}$

Prime $_{j+1}$ – Prime $_{j}$ = [ ($P_j$*)² / ( Pj*)² – 1]$^{1/(1-t)}$ - [ ($P_{j-1}$*)² / ( $P_{j-1}$*)² – 1]$^{1/(1-t)}$

Prime $_{j+1}$ – Prime $_{j}$ = { [($P_j$*)²- 1] + 1 / [($P_j$*)² - 1]}$^{1/(1-t)}$ - { [($P_{j-1}$*)² - 1] + 1 / [($P_{j-1}$*)² - 1]}$^{1/(1-t)}$

= [1 + (1/(1-t))*(1/[($P_j$*)² - 1])] - [1 + (1/(1-t))*(1/[($P_{j-1}$*)² - 1])]

= (1/(1-t))*[($P_{j-1}$*)² – ($P_j$*)²] / {[( $P_j$*)² – 1]*[( $P_{j-1}$*)² – 1]}

= [(1/(1-t))*(-1/j)*(2$P_{j-1}$* + 1/j)] / {[($P_j$*)²($P_{j-1}$*)² – [($P_j$*)2+ ($P_{j-1}$*)² ] + 1}

i.e. (((1-t) ) / |(1-t)|2)*(-1/j)*(2Pj-1* + 1/j)] / {[(Pj*)2(Pj-1*)2 – [(Pj*)2+ (Pj-1*)2 ] + 1},

Alternatively, we may consider:

(Prime j+1)(1-t) – (Prime j)(1-t) = { [(Pj*)2 - 1] + 1 / [(Pj*)2 - 1]} - { [(Pj-1*)2 - 1] + 1 / [(Pj-1*)2 - 1]}

As (Prime j+1)(1-t) = [1 + (Prime j+1 – 1)](1-t)

= 1 + (1-t)*(Prime j+1 – 1),

thus (Prime j+1)(1-t) – (Prime j)(1-t) = (1-t)*(Prime j+1 - Prime j)

= [(Pj-1*)2 – (Pj*)2] / {[( Pj*)2 – 1]*[( Pj-1*)2 – 1]},

Prime j+1 - Prime j = [(1/(1-t))*(-1/j)*(2Pj-1* + 1/j)] / {[(Pj*)2(Pj-1*)2 – [(Pj*)2+ (Pj-1*)2 ] + 1}

But P*j+1 = [Pj+1]*W & s = (1-t)+vi,

Prime j+1 – Prime j = {[-(¯((1-t) ) / |(1-t)|2)*(1/j)] (2W*Pj-1 + (1/j))]} / [W4(Pj)2(Pj-1)2 – W2[(Pj)2 + (Pj-1)2] + 1], or two methods produce the same expression result for the prime gap.

In terms of s = (1-t) +v*I, we may get:

Prime j+1 - Prime j = [(1/(1-t))*(-1/j)*(2Pj-1* + 1/j)] / {[(Pj*)2(Pj-1*)2 – [(Pj*)2+ (Pj-1*)2 ] + 1} -------- (******)

Now suppose s = (1+t) + v*I, we may still get: (where 0 < t <1)

Prime j+1 - Prime j

= [(1/(1+t))*(-1/j)*(2Pj-1* + 1/j)] / {[(Pj*)2(Pj-1*)2 – [(Pj*)2+ (Pj-1*)2 ] + 1}

When t → 0,

Prime j+1 - Prime j = [(-1/j)*(2Pj-1* + 1/j)] / {[(Pj*)2(Pj-1*)2 – [(Pj*)2+ (Pj-1*)2 ] + 1}

= [(-1/j)*(2Pj-1* + 1/j)] / {[(Pj*)2(Pj-1*)2 – [2*(Pj-1*)2 + 2 (1/j)*(Pj-1*) + (1/j)2]

= [(-1/j)*(2Pj-1* + 1/j)] / {[(Pj*)2(Pj-1*)2 – [2*(Pj-1*) (Pj-1* - (1/j)) + (1/j)2]

When Pj-1* = (1/j),

Prime j+1 - Prime j = [(-1/j)*(2Pj-1* + 1/j)] / {[(Pj*)2(Pj-1*)2 - (1/j)2}

= -3(1/j)2 / {[(Pj*)(Pj-1*) - (1/j)][(Pj*)(Pj-1*) + (1/j)]}

= -3 (1/j)2 / [4(1/j)4 – (1/j)2]

$= -3 / \left[4(1/j)^2 - 1\right] .\leq -\frac{3}{4\left(\frac{1}{j}\right)^2} .. \leq -\frac{3}{4}$ when j → 1

But when j → ∞, $\text{Prime}_{j+1}$ - $\text{Prime}_j$ = 3.

i.e. $\text{Prime}_{j+1}$ - $\text{Prime}_j$ .⩽ -3/4 and $\text{Prime}_{j+1}$ - $\text{Prime}_j$ .⩽ 3

Or for the case s = 1 ± t, $\text{Prime}_{j+1}$ - $\text{Prime}_j$ .⩽ -3/4 when t → 0, which is obviously a contradiction to the fact that the prime gap should be greater than zero and tends to a very large value but NOT a fractional value. This is because of the assumption to the all kinds of Riemann Zeta equivalent function equations will also be satisfied by the ξ(1) which is divergent as listed in the beginning. In fact, when the de-nominator attains its minimum, then the prime gap gets its maximum value, hence, we differentiate the above implicit equations by Chain Rule and set the result equals to zero for the minimum values,

$$\frac{\partial f(w)}{\partial k} = \frac{\partial f(w)}{\partial w} * \frac{\partial w}{\partial k}$$

$f(W) = W^4 * P_j^2 * P_{j-1}^2 - W^2 * \left[P_j^2 + P_{j-1}^2\right] + 1, \frac{\partial f(w)}{\partial w}$, this gives us:

$= \left\{4W^3 * \left[\left(P_j^2\right)\left(P_{j-1}^2\right)\right] - 2W * \left[P_j^2 + P_{j-1}^2\right]\right\} \frac{\partial w}{\partial k}$

Also, we need to approximate W in (*"*) by Taylor series and then differentiate W with respect to k (or replaced by x) and taking the limit u → 1-t in the mathematical computation, we may finally have:

$\text{Prime}_{j+1} - \text{Prime}_j$ = {[-1 / ((1-t) + vi)*j]* (1/j)]}

= {[-1 / ((1-t) + vi)*j]* (1/j)]}

Hence, the computed prime gap is obviously in a complex number format but the result should theoretically be an integer or greater than zero.

In fact, for

$1/\text{Prime}_{j+1} = [1 - 1/(P_j^*)^2]^{1/(1-t)}$, $\text{Prime}_{j+1} = [(P_j^*)^2/((P_j^*)^2-1)]^{1/(1-t)}$

$\ln(\text{Prime}_{j+1}) = [1/(1-t)] \ln [(Pj^*)2/((P_j^*)^2-1)]$

$\ln(\text{Prime}_j) = [1/(1-t)] \ln [(P_{j-1}^*)^2/((P_{j-1}^*)^2-1)]$

$\ln(\text{Prime}_{j+1}) - \ln(\text{Prime}_j) = [1/(1-t)] [\ln (P_j^*)^2 - \ln ((P_j^*)^2-1)] - [1/(1-t)] [\ln (P_{j-1}^*)^2 - \ln ((P_{j-1}^*)^2-1)]$

$= 1/(1-t) \{\ln [(P_j^*)^2/((P_j^*)^2-1)] - \ln [(P_{j-1}^*)^2/((P_{j-1}^*)^2-1)]\}$

$\ln [(\text{Prime}_{j+1})/(\text{Prime}_j)] = 1/(1-t) \ln [(P_j^*)^2/((P_j^*)^2-1) . ((P_{j-1}^*)^2-1)/(P_{j-1}^*)^2]$

$.\leq 1/(1-t) \ln [((P_j^*)^2-1)/((P_j^*)^2-1) . (P_j^*)^2/(P_{j-1}^*)^2]$

$.\leq 1/(1-t) \ln (P_j^*)^2/(P_{j-1}^*)^2 = \ln (1/j)^{2/(1-t)}$

When j→∞, 1/j→0, hence $[(\text{Prime}_{j+1})/(\text{Prime}_j)] .\leq .0$.

Thus, we have shown that the prime gap for the case of the index power s is smaller than one. However, in the above first proof for s = (1-t) + vi, we have just shown that for j tends to infinity, the prime gap tends to zero. But actually the prime gap should be greater than zero. Therefore, the contradiction occurs due to the assump tion that $(1-2^{1-(1-t)})\xi(1-t)$ satisfies all of the Riemann Zeta equations (*****). Actually, the prime gap is independent of the integral ZFC, we may need to check whether there may be a continuum structure by the concept of forcing that uses countable transitive models – ctm – of ZFC or the disproof counter example to the continuum as shown in my amended master thesis, 2015. Thus, this author may propose that there may be some

number system rather than ZFC where the Riemann Hypothesis should be true and some other number system that the Riemann Hypothesis should be false. Or there may need to have a 3-dimensional complex number system which is impossible! Instead, we may have the n-dimensional complex vector space. By taking u = 1-t, we may have:

As $W=\frac{\left(\left[(1-t)-v*cot(x+1)\right]\right)}{\left(r*\left[(1-t)*cot(x+1)+v\right]\right)}$,

$$\frac{\partial W}{\partial x}=\frac{\left(\left[-v*\left(-1-cot(x+1)^2\right)\right]\right)}{\left(r*\left[(1-t)*cot(x+1)+v\right]\right)}-\frac{\left[(1-t)-v*cot(x+1)\right]*[0]+r*\left[(1-t)*\left(-1-cot(x+1)^2\right)\right]}{r*\left[(1-t)*cot(x+1)+v\right]^2}$$

$$=\frac{\left(\left[-v*(-1)csc^2(x+1)\right]\right)*\left(r*\left[(1-t)*cot(x+1)+v\right]\right)-\left[(1-t)-v*cot(x+1)\right]*\left[[0]+r*\left[(1-t)*\left(-1-cot(x+1)^2\right)\right]\right]}{r*\left[(1-t)*cot(x+1)+v\right]^2}$$

$$=\frac{\left(\left[-v*(-1)*csc^2(x+1)\right]\right)*\left(r*\left[(1-t)*cot(x+1)+v\right]\right)-\left[(1-t)-v*cot(x+1)\right]*r*\left[(1-t)*(-1)*csc^2(x+1)\right]}{r*\left[(1-t)*cot(x+1)+v\right]^2}$$

$$=csc^2(x+1)\frac{\left[\left[v\left(r\left(ucot(x+1)+v\right)\right)\right]-\left[u-vcot(x+1)\right]*(-ru)\right]}{r*\left[u*cot(x+1)+v\right]^2}$$

$$=\frac{r\left((1-t)^2+v^2\right)csc^2(x+1)}{r*\left[(1-t)*cot(x+1)+v\right]^2}=\frac{\left((1-t)^2\right)csc^2(x+1)}{\left[(1-t)*cot(x+1)+v\right]^2}$$

For $\partial W/\partial x = 0$, we may have $\csc^2(x+1) = 0$ or $\cot^2(x+1) = -1$

i.e. $\cot(x+1) = \pm I$. And by continuing the solving procedure, we get:

If x = 1/2, then $\text{Prime}_{j+1} - \text{Prime}_j = \frac{\left(\frac{1}{s}\right)\left(\frac{1}{j}\right)\left((2)\left(\frac{1}{\sqrt{2}}\right)\left(P_{j-1}\right)+\left(\frac{1}{j}\right)\right)}{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)P_j^2P_{j-1}^2-\left[\frac{1}{2}P_j^2+\frac{1}{2}P_{j-1}^2\right]+1}$

By taking limit j tends to infinity and $(1 / 1 + l(j)) \leq Pj \leqslant (1 / ln(j))$,

$[W^4(P_j)^2(P_{j-1})^2 - W_2[(P_j)^2 + (P_{j-1})^2] + 1]$ tends to 1 and $2W^* P_{j-1}$ tends to zero.

The final result of the prime gap will still be a complex number as the prime gap equal to:

$\text{Prime}_j - \text{Prime}_{j-1} = \{[-1 / ((1-t)+vI)^*j]^* (1/j)]\} = \frac{-1}{((1-t)+vI)}$

which is still a complex number or a contradiction. There is a similar outcome for x = -1/2, this author will NOT repeat.

## A Fine Adjustment to the Riemann Zeta Root Model Equation

It is no doubt that this author have already found a Riemann Zeta (Function)'s Root Model Equation -- $0.5\pm\frac{4cot(ln(x))}{(x+1)^2}i.$. However, the root model equation subjects to the deficiencies of the essential coefficients and is just an empirical one. In reality, convex optimization is famous for its usage in the filter design as well as the feedback controller but the focus of the present paper is not in the research of digital signal processing or electronic engineering. Hence, we may obtain the respective coefficients for the present empirical Riemann Zeta (Function)'s Root Model Equation -- $0.5\pm\frac{4cot(ln(x))}{(x+1)^2}i.$

The following are the practically results (with a brief description for the procedure or how) that may be gained from the licensed Maple Soft Personal Edition 2022:

### Manually "Try & Error" Method For the Convergence

First, we may use "fsolve" command in Maple to find the convergence between the zeta root model equation $0.5\pm\frac{4cot(ln(x))}{(x+1)^2}i$ and the first ten non-trivial zeta root zeros polynomial – $0.0004319x^9 - 0.02216x^8 + 0.4869x^7 - 5.986x^6 + 45.1x^5 - 214.5x^4 + 638.4x^3 - 1134x^2 + 1080x - 395.2$

input: $\text{fsolve}\left(\left(\left(\frac{4cot\left(\frac{2(x-1)}{(x+1)}\right)}{0.05(x+1)^2}\right)-0.0004319x^9-0.02216x^8+0.4869x^7-5.986x^6+45.1x^5-214.5x^4+638.4x^3-1134x^2+1080x-395.2\right),x,0,infinity\right)$

output: 0.7948976085

input: $\text{fsolve}\left(\left(\left(\frac{4cot\left(\frac{2(x-1)}{(x\ 1)}\right)}{0.05(x+1)^2}\right)-0.0004319x^9-\ 0.02216x^8+\ 0.4869x^7-\ 5.986x^6+\ 45.1x^5-\ 214.5x^4+\ 638.4x^3-\ 1134x^2+\ 1080x-395.2\right),x,\ 0,\ infinity\right)$

output: 0.5618354841

input: $\text{fsolve}\left(\left(\frac{4cot\left(\frac{2(x-1)}{(x+1)}\right)}{0.000001024(x+1)^2}\right)-(0.0004319x^9-\ 0.02216x^8+\ 0.4869x^7-\ 5.986x^6+\ 45.1x^5-\ 214.5x^4+\ 638.4x^3-\ 1134x^2+\ 1080x-395.2),\ x,\ 0,\ infinity\right)$

output: 0.1202265952

We observe that the difference between the zeta root model equation and the first ten zeta zeros polynomial or the convergence drops dramatically from 0.7 to 0.1 when the coefficient of the term $(x + 1)^2$ approaches to zero. In other words, the coefficient of $(x + 1)^2$ may be the source of discrepancy. The larger the value of the coefficient in $(x + 1)^2$, the greater will be the discrepancy. Indeed, the coefficient of $(x + 1)^2$ is just the source of error to the convergence between these two equations or the empirical equation $\frac{4}{(x+1)^2}cot\left(\frac{2(-1)}{(x+1)}\right)$ We may finely adjust the convergence error by adding a significantly small value as the coefficient of $(x+ 1)^2$ to greatly reduce the discrepancy. i.e. One of the feasible zeta root model equation may be: $0.5\pm\frac{4cot(ln(x))}{\delta(x+1)^2}i$ where δ tends to zero. Or the limit of $\frac{4cot(ln(x))}{(x+1)^2}$ is just the (first ten) non-trivial zeta roots polynomial. That says,

$$\lim_{x\to\infty}\frac{4cot(ln(x))}{(x+1)^2}=0.0004319x^9-\ 0.02216x^8+\ 0.4869x^7-\ 5.986x^6+\ 45.1x^5-\ 214.5x^4+\ 638.4x^3-\ 1134x^2+\ 1080x-\ 395.2$$

This limit implies that $\frac{4cot(ln(x))}{(x+1)^2}$ is just an empirical equation for the imaginary part of the zeta root model equation or may be the best model equation under a certain constrained conditions. Thus we may assume that the true root model equation should be:

$$\frac{\beta cot(ln(x))}{(\alpha x+1)^n}$$

In fact, we may employ the similar convergence method to optimize the above zeta root model equation such that the difference between $\frac{\beta cot(ln(x))}{(\alpha x+1)^n}$ and the (first ten) non-trivial zeta interpolation polynomial tends to be a minimum (or maximum) etc. Practically, we may apply the Lagrange Multiplier method to solve the above prescribed optimization problem. In reality, one of the application for the present optimization is in the field of biological & medicinal man-made vaccine development (or the optimal mutation point for the feasible virus such as COVID-19 & influenza evolution etc) which is an interesting issue. The following described may act as an important procedure and reference for the above artificial vaccine making.

## Manually (Trial & Error) Optimization Tuning

Next, we may proceed to the optimization for the difference between the Zeta root model equation and the first ten roots' interpolation polynomial. This author will apply the optimization method (or the Lagrange Multiplier way) of the Maple Soft to fine tune the root model equation. At the same time, this author will also employ the manual bisectional adjustment method in the search scheme for a primal or dual (maximum or minimum) sets of coefficients.

In reality, by continuing multiply a factor of 2n to the inner coefficient equals to value of 126 for the term cotangent, we may get the minimum of x equals to 0.128 by the method of trial and error:

$$Optimization[Minization]\left[\frac{4}{(x+1)^2}*cot\left(\frac{(x-1)}{(x+1)}\right)-\left(\frac{7}{17280}*x^9-\frac{167}{8064}*x^8+\frac{9133}{20160}*x^7-\frac{15953}{2880}*x^6+\frac{238957}{5760}*x^5-\frac{225833}{1152}*x^4+\frac{313033}{540}*x^3-\frac{1146773}{1120}*x^2+\frac{816371}{840}*x-354\right)\right],x=-50..1$$

Answer: [$-1.16847467496693*10^8$, [x = 0.999999982883666]]

$$Optimization[Minization]\left[\frac{4}{(x+1)^2}*cot\left(\frac{64(x-1)}{(x+1)}\right)-\left(\frac{7}{17280}*x^9-\frac{167}{8064}*x^8+\frac{9133}{20160}*x^7-\frac{15953}{2880}*x^6+\frac{238957}{5760}*x^5-\frac{225833}{1152}*x^4+\frac{313033}{540}*x^3-\frac{1146773}{1120}*x^2+\frac{816371}{840}*x-354\right)\right],x=-50..1$$

Answer: [$-3.51843720888460*10^{13}$, [x = 1.]]

$$Optimization[Minization]\left[\frac{4}{(x+1)^2}*cot\left(\frac{96(x-1)}{(x+1)}\right)-\left(\frac{7}{17280}*x^9-\frac{167}{8064}*x^8+\frac{9133}{20160}*x^7-\frac{15953}{2880}*x^6+\frac{238957}{5760}*x^5-\frac{225833}{1152}*x^4+\frac{313033}{540}*x^3-\frac{1146773}{1120}*x^2+\frac{816371}{840}*x-354\right)\right],x=-50..1$$

Answer: [$-4.69124961184567*10^{13}$, [x = 1.]]

$$Optimization[Minization]\left[\frac{4}{(x+1)^2}*cot\left(\frac{103(x-1)}{(x+1)}\right)-\left(\frac{7}{17280}*x^9-\frac{167}{8064}*x^8+\frac{9133}{20160}*x^7-\frac{15953}{2880}*x^6+\frac{238957}{5760}*x^5-\frac{225833}{1152}*x^4+\frac{313033}{540}*x^3-\frac{1146773}{1120}*x^2+\frac{816371}{840}*x-354\right)\right],x=-50..1$$

Answer: [-2.14458989343445*10^6, [x = 0.885010932207843]]

$$Optimization[Minization]\left[\frac{4}{(x+1)^2}*cot\left(\frac{120(x-1)}{(x+1)}\right)-\left(\frac{7}{17280}*x^9-\frac{167}{8064}*x^8+\frac{9133}{20160}*x^7-\frac{15953}{2880}*x^6+\frac{238957}{5760}*x^5-\frac{225833}{1152}*x^4+\frac{313033}{540}*x^3-\frac{1146773}{1120}*x^2+\frac{816371}{840}*x-354\right)\right],x=-50..1$$

Answer: [$-1.96689921175741*10^6$, [x = 0.810413891279105]]

$$Optimization[Minization]\left[\frac{4}{(x+1)^2}*cot\left(\frac{126(x-1)}{(x+1)}\right)-\left(\frac{7}{17280}*x^9-\frac{167}{8064}*x^8+\frac{9133}{20160}*x^7-\frac{15953}{2880}*x^6+\frac{238957}{5760}*x^5-\frac{225833}{1152}*x^4+\frac{313033}{540}*x^3-\frac{1146773}{1120}*x^2+\frac{816371}{840}*x-354\right)\right],x=-50..1$$

Answer: [-2.70699098327853*10$^6$, [x = 0.128075145828693]]

$$Optimization[Minization]\left[\frac{4}{(x+1)^2}*cot\left(\frac{128(x-1)}{(x+1)}\right)-\left(\frac{7}{17280}*x^9-\frac{167}{8064}*x^8+\frac{9133}{20160}*x^7-\frac{15953}{2880}*x^6+\frac{238957}{5760}*x^5-\frac{225833}{1152}*x^4+\frac{313033}{540}*x^3-\frac{1146773}{1120}*x^2+\frac{816371}{840}*x-354\right)\right],x=-50..1$$

Answer: [-108189.944004466, [x = 0.781390351828491]]

Obviously, x attains its minimum at 126 and then rise its value again.

Then we may compare the above empirical Riemann Zeta Function model equation $\frac{4cot(ln(x))}{(x+1)^2}$ with the case $\frac{4cot(ln(x))}{(x+1)^3}$, we may have:

$$Optimization[Minization]\left[\frac{4}{(x+1)^3}*cot\left(\frac{4(x-1)}{(x+1)}\right)-\left(\frac{7}{17280}*x^9-\frac{167}{8064}*x^8+\frac{9133}{20160}*x^7-\frac{15953}{2880}*x^6+\frac{238957}{5760}*x^5-\frac{225833}{1152}*x^4+\frac{313033}{540}*x^3-\frac{1146773}{1120}*x^2+\frac{816371}{840}*x-354\right)\right],x=-50..1$$

Answer: [-6.79657163760862*10$^6$, [x = 0.999999963216672]]

$$Optimization[Minization]\left[\frac{4}{(x+1)^3}*cot\left(\frac{256(x-1)}{(x+1)}\right)-\left(\frac{7}{17280}*x^9-\frac{167}{8064}*x^8+\frac{9133}{20160}*x^7-\frac{15953}{2880}*x^6+\frac{238957}{5760}*x^5-\frac{225833}{1152}*x^4+\frac{313033}{540}*x^3-\frac{1146773}{1120}*x^2+\frac{816371}{840}*x-354\right)\right],x=-50..1$$

Answer: [-743105.944465381, [x = 0.952088533573994]]

$$Optimization[Minization]\left[\frac{4}{(x+1)^3}*cot\left(\frac{1024(x-1)}{(x+1)}\right)-\left(\frac{7}{17280}*x^9-\frac{167}{8064}*x^8+\frac{9133}{20160}*x^7-\frac{15953}{2880}*x^6+\frac{238957}{5760}*x^5-\frac{225833}{1152}*x^4+\frac{313033}{540}*x^3-\frac{1146773}{1120}*x^2+\frac{816371}{840}*x-354\right)\right],x=-50..1$$

Answer: [-1842.71213128505, [x = 0.684712613636625]]

$$Optimization[Minization]\left[\frac{4}{(x+1)^3}*cot\left(\frac{2048(x-1)}{(x+1)}\right)-\left(\frac{7}{17280}*x^9-\frac{167}{8064}*x^8+\frac{9133}{20160}*x^7-\frac{15953}{2880}*x^6+\frac{238957}{5760}*x^5-\frac{225833}{1152}*x^4+\frac{313033}{540}*x^3-\frac{1146773}{1120}*x^2+\frac{816371}{840}*x-354\right)\right]$$

Answer: [-1.94156446897192*10$^6$, [x = 0.158005915456556]]

It seems that when the inner multiply coefficient of the cotangent equals to 2048, the value of x will attain its minimum at 0.158 together with y = -1.94156*10$^6$. In practice, both of the empirical Riemann Zeta function's root model equations have similar paired values of minimum but the factor for the inner coefficient of the cotangent is much larger for the case of $\frac{4cot(ln(x))}{(x+1)^3}$. This author thus proposes that both $0.5\pm\frac{4cot(126*(ln(x)))}{(x+1)^2}I$ and $0.5\pm\frac{4cot(2048*(ln(x)))}{(x+1)^3}I$ are two of the infinite many Riemann Zeta function's root of optimized model equations.

(N.B. Certainly, we may also have the so-called business simplex method, the dual part of optimization for a maximization of the aforementioned two optimized equations which is out the focus of the present research.)

## An Interested Elliptical Discriminant for the Riemann Hypothesis Inequality

Consider the following Riemann Hypothesis Number Theory Equation:

$$\sum_\rho \frac{x^\rho}{\rho}+ln(2\pi)+\frac{1}{2}ln\left(1-x^{-2}\right)=0$$

$$\left|\left(\sum_\rho \frac{x^\rho}{\rho}\right)\right|=\left|-\left(ln(2\pi)+\frac{1}{2}ln\left(1-x^{-2}\right)\right)\right|$$

$$=ln\left(2\pi\sqrt{1-\frac{1}{x^2}}\right)$$

In addition, $\left|\left(\psi(x)-x\right)\right|=ln\left(2\pi\sqrt{1-\frac{1}{x^2}}\right)$ (1)

According to Schoenfeld in 1976, we may have:

$$\frac{\left|\left(\psi(x)-x\right)\right|}{\left|\left(\pi(x)-li(x)\right)\right|}<\frac{\frac{1}{8\pi}\sqrt{x}log^2(x)}{\frac{1}{8\pi}\sqrt{x}log(x)}=log(x)$$

Or we may have:

$$\left|\left(\pi(x)-li(x)\right)\right|=\frac{\left|\left(ln\left(2\pi\sqrt{1-\frac{1}{x^2}}\right)\right)\right|}{log(x)-M_1}$$ (2)

$$\frac{1}{8\pi}\sqrt{x}log^2(x)-\left|\left(\psi(x)-x\right)\right|+M_2=0$$ (3)

$$\frac{1}{8\pi}\sqrt{x}log(x)-\left|\left(\pi(x)-li(x)\right)\right|+M_3=0 \qquad (4)$$

(3) – (4), we may have to consider the following quadratic equation:

$$\frac{1}{8\pi}\sqrt{x}log^2(x)-\frac{1}{8\pi}\sqrt{x}log(x)-\left|\left(\psi(x)-x\right)\right|+\left|\left(\pi(x)-li(x)\right)\right|+M_2-M_3=0$$

Substituting (1) & (2) into the above equation, we get:

$$\frac{1}{8\pi}\sqrt{x}log^2(x)-\frac{1}{8\pi}\sqrt{x}log(x)-ln\left(2\pi\sqrt{1-\frac{1}{x^2}}\right)+\frac{\left|\left(ln\left(2\pi\sqrt{1-\frac{1}{x^2}}\right)\right)\right|}{log(x)-M_1}+M_2-M_3=0$$

Or

$$\frac{1}{8\pi}\sqrt{x}log^3(x)-\frac{1}{8\pi}\sqrt{x}log^2(x)-ln\left(2\pi\sqrt{1-\frac{1}{x^2}}\right)log(x)+ln\left(2\pi\sqrt{1-\frac{1}{x^2}}\right)-\frac{1}{8\pi}M_1\sqrt{x}log^2(x)+$$

$$\frac{1}{8\pi}M_1\sqrt{x}log(x)+M_1ln\left(2\pi\sqrt{1-\frac{1}{x^2}}\right)+(M_2-M_3)(log(x)-M_1)=0$$

Then, this author employs the Maple Soft to approximate (as some terms may be neglected) and optimize the above equation by:

$$Optimization[Minimize]\left(\sqrt{(x)}*\frac{ln(x)^3}{(8*Pi*ln(10)^3)}-\sqrt{(x)}*\frac{ln(x)^2}{(8*Pi*ln(10)^2)}-ln(2*(M+1)*Pi)*log(x)+ln(2*(M+1)*Pi),x=1..5000\right)$$

We get the point:

(x = 1162.48662088001, y = -129.806423440223853, M = $2.48004159219448*10^{10}$)

such that the equation (3) meets the equation (4) at that (optimized) minimum point. But y is a negative value (an abnormal result) which may imply that there may be a possibility both of the Schoenfeld inequality for the Riemann Hypothesis will fail. This is because the conditional subjection end-point or the critical one may be approximately equal to x = 1162 but neither what the inequalities may suggest to be x > 2657 nor x > 73.2. Therefore, this author proposes that the above case may be considered as a counter example for the disproof to the Riemann Hypothesis.

To be precise, one may also find the inflection point for the optimization (maximize the minimum) of the following equation:

$$\sqrt{(x)}*\frac{ln(x)^3}{(8*Pi*ln(10)^3)}-\frac{ln(x)^2*\sqrt{(x)}}{(8*Pi*ln(10)^2)}-ln((M+1)*Pi)*log(x)+ln((M+1)*Pi)$$
$$-k*\sqrt{(x)}*\frac{ln(x)^2}{(8*Pi*ln(10)^2)}+k*\sqrt{(x)}*\frac{ln(x)}{(8*Pi*ln(10)^{\%H})}$$
$$+k*ln((M+1)*Pi)+(l-j)(log[10](x)-k)$$

When we select l = 320, j = 160 & k = 6.4725, then we may find the corresponding point:

$$Optimiation[Minimize]\left(\left(\left(\sqrt{(x)}*\frac{\ln(x)^3}{(8*pi*\ln(10)^3)}-\frac{\ln(x)^2*\sqrt{(x)}}{(8*pi*\ln(10)^2)}-\ln((M+1)*Pi)*\log(x)+\ln((M+1)*Pi)\right)\right.\right.$$
$$\left.\left.-\frac{(6.4725*\ln(x)^2*\sqrt{(x)})}{8*pi*\ln(10)^2}+\frac{(6.4725*\sqrt{(x)}*\ln(x))}{8*pi*\ln(10)}\right)$$
$$+6.4725*ln((M+1)*Pi)$$
$$-(320-160)(\log[10](x)-6.4725),x=1..3516\Bigg)$$

where a complex valued answer will be encountered.

However, if we set a small ε to x = 3516 + 0.125, then we may get the wanted (approximate) critical point for a real answer:

$$Optimiation[Minimize]\left(\left(\left(\sqrt{(x)}*\frac{\ln(x)^3}{(8*pi*\ln(10)^3)}-\frac{\ln(x)^2*\sqrt{(x)}}{(8*pi*\ln(10)^2)}-\ln((M+1)*Pi)*\log(x)+\ln((M+1)*Pi)\right)\right.\right.$$
$$\left.\left.-\frac{(6.4725*\ln(x)^2*\sqrt{(x)})}{8*pi*\ln(10)^2}+\frac{(6.4725*\sqrt{(x)^2}*\ln(x))}{8*pi*\ln(10)}\right)$$
$$+6.4725*ln((M+1)*Pi)$$
$$-(320-160)(\log[10](x)-6.4725),x=1..3516.125\Bigg)$$

y = -239.989780089386244, M = $3.72956556322979*10^{10}$, x = 3516.125

In other words, for all x before the (approximated) value x = 3516(.125), then the answer will be a complex value. But if we adjust it with an ε = 0.125, then the equation will give a negative y which is the largest minimum point. After that critical point, y will continue decrease as lowest as possible until the negative infinity. From the above individual case, we observe that there may be infinite many critical points corresponding to infinite different paired values of the constant variables in the equation.

***In general, this writer conclude that:***

*"We may approximate each of the (infinitely many) critical points by the above Maple Soft optimization minimizing method (and the procedure) together with the addition of the epsilon ε sufficiently small value to obtain the largest minimum point y or the wanted infinite number of critical points."*

That is, the general solution to the above Elliptical Discriminant for the Riemann Hypothesis Inequality should be:

1. Let

$$y=\frac{1}{8\pi}\sqrt{x}log^3(x)-\frac{1}{8\pi}\sqrt{x}log^2(x)-ln\left(2\pi\sqrt{1-\frac{1}{x^2}}\right)log(x)+ln\left(2\pi\sqrt{1-\frac{1}{x^2}}\right)-$$

$$\frac{1}{8\pi}M_1\sqrt{x}log^2(x)+\frac{1}{8\pi}M_1\sqrt{x}log(x)+M_1ln\left(2\pi\sqrt{1-\frac{1}{x^2}}\right)+(M_2-M_3)(log(x)-M_1)$$

2. General solution to y is:

{y = 0} + (one of a particular solutions which is equal to -239.989780089386244)

For y = 0

- $x_1$ = 2657 if we assume the Riemann Hypothesis is true

or

- $x_3$ = 1162 if we assume the Riemann Hypothesis is false;

For the particular solution, y = -239.98780089386244, $x_2$ = 3516.125 and the difference between

$x_2 - x_1$ = 859.125 if we assume the case (a)

For the particular solution, y = -239.98780089386244, $x_2$ = 3516.125 and the difference between

$x_2 - x_3$ = 2354.125 if we assume the case (b)

**As the General Solution = Homogeneous Solution + Particular Solution, thus we may have:**

Assume case (a)

$$\begin{pmatrix}x_1\\0\end{pmatrix}+\begin{pmatrix}x_2-x_1\\y\end{pmatrix}=\begin{pmatrix}2657\\0\end{pmatrix}+\begin{pmatrix}859.125\\-239.98780089386244\end{pmatrix}$$ = Solution 1

or

$$\begin{pmatrix}x_1\\0\end{pmatrix}+\begin{pmatrix}x_2\\y\end{pmatrix}=\begin{pmatrix}2657\\0\end{pmatrix}+\begin{pmatrix}3516.125\\-239.98780089386244\end{pmatrix}$$ = Solution 2

or

$$\begin{pmatrix}x_1\\0\end{pmatrix}+\begin{pmatrix}x_2+(x_2-x_1)\\y\end{pmatrix}=\begin{pmatrix}2657\\0\end{pmatrix}+\begin{pmatrix}3516.125+859.125\\-239.98780089386244\end{pmatrix}$$ = Solution 3

or

$$\begin{pmatrix}x_1\\0\end{pmatrix}+\begin{pmatrix}x_1-x_2\\y\end{pmatrix}=\begin{pmatrix}2657\\0\end{pmatrix}+\begin{pmatrix}2657-3516.125\\-239.98780089386244\end{pmatrix}$$ = Solution 4

***Assume case (b)***

$$\begin{pmatrix}1162\\0\end{pmatrix}+\begin{pmatrix}2354.125\\-239.98780089386244\end{pmatrix}$$ = Solution 5

or

$$\begin{pmatrix}1162\\0\end{pmatrix}+\begin{pmatrix}3516.125\\-239.98780089386244\end{pmatrix}$$ = Solution 6

or

$$\begin{pmatrix}1162\\0\end{pmatrix}+\begin{pmatrix}3516.125+2354.125\\-239.98780089386244\end{pmatrix}$$ = Solution 7

or

$$\begin{pmatrix}1162\\0\end{pmatrix}+\begin{pmatrix}1162-2354.125\\-239.98780089386244\end{pmatrix}$$ = Solution 8

In brief, we may summarize the general solutions as below:

$$\begin{pmatrix} x_h \\ 0 \end{pmatrix} + \begin{pmatrix} \pm(x_p - x_h) \\ y_p \end{pmatrix}$$

$$\begin{pmatrix} x_h \\ 0 \end{pmatrix} + \begin{pmatrix} (x_p) \\ y_p \end{pmatrix}$$

$$\begin{pmatrix} x_h \\ 0 \end{pmatrix} + \begin{pmatrix} (x_p + (x_p - x_h)) \\ y_p \end{pmatrix}$$

where h means the two sets of homogeneous solution for either RH is true or RH is false and p means a particular solution.

Certainly, in the mirror image reverse way, for any given critical point such as the case x = 2657, one may find back the paired variables conditional y's value which may be a complex value or a real number etc. Thus, the implication is, we may view the above outcome as the counting of lattice point(s) for the traveling sale-person problem or the elliptical function problems in the number theory for the (quantum) lattice cryptography etc. Certainly, the mirror image inverse is also true but one should be aware of the case one divided by zero which is an infinity. In such of the situation, one may need to apply the duality of the optimization method (maximize the minimum / minimize the maximum by Maple Soft) for solving such problems. To go ahead a step, for the equation 1/x when x tends to zero, one may then consider the maximize the minimum of y = [1/ (1/x)] and hence restore back 1/x by y's optimization duality etc or the vice versa is also true. The following is a case study for the Maple Soft script:

t1 := 1/r6;

**output:** **t1 := 1/r$^6$**

t1_numeric := convert(t1, polynom);

**output:** **t1_numeric := 1/r$^6$**

f := proc(t_val) return evalf(eval(1/t1_numeric, [t1 = t_val, r = infinity])); end proc;

Optimization[Maximize](f, 0 .. 1);

Warning, limiting number of evaluations reached

**output:** **[Float(infinity), [0.199999504502172]]**

Thus, by the primal-duality properties of the optimization, the (maximized) minimum value of r6 is just 0.199999504502172 or approximately {min[max] (f = 1/r$^6$)} = 5 ----- (*****). But as the roots of an elliptical curve are usually located on the two sides of the minimum with equal distance or $_{min=\frac{r_1+r_2}{\partial}}$, then by the sense of commercial reverse engineering (an optimization to the statistical principal component analysis etc), one may actually compute back those values of r1 and r2 etc. In reality, the aforementioned method can therefore be applied in the field of cryptography for encryption and decryption. In practice, with the use of Hessian matrix and its determinant, Jensen's Inequality, multivariable analysis, one may find the point at infinity together with the 8 affine points. By applying the Gallant Lambert Vanstone Decomposition, one may split 256 bit multiplication kp into two parts:

$$kp = k_1 p + k_2 \phi(p)$$

With the use of both primitive cube root β and the multi-scalar multiplication technique to the hardware, one may speed up the calculation of both $k_1 p$ and $k_2 \phi(p)$ and hence we can solve the $\psi_3(x) = 0$ for a specific Bitcoin curve like secp256k1.

(N.B. Optionally, one may go a forward step for the construction of the elliptic functions together with the Weierstrass P function in the form of:

$$P(z) = \frac{1}{x^2} + \frac{(g_2)x^2}{20} + \frac{(g_3)x^4}{28} + \frac{(g_2^2)x^6}{1200} + \frac{(3g_2g_3)x^8}{6160} + \frac{\left((49g_2^3 + 750g_3^2)x^{10}\right)}{7644000} + \frac{(g_2^2 g_3)x^{12}}{184800} + O[x]^{13}$$

In a simple and informal sense, there is a one-one correspondence between the elliptical curves and elliptical functions. Hence, we may employ all of the integral points to the elliptical curves as the initial starting period and compute the necessary respective gi s or the discriminant(s) of the Weierstrass P function for the elliptical function(s) with respective to the suitable modular division or arithmetic. Actually, the focus lays in how may we select a suitable elliptical curve (or the parameter in the family of such elliptical curve) so as to obtain the best optimization of the Weierstrass P function for the elliptical function lattice. In fact, when the elliptical function's lattice parameter(s) become(s) optimized, then the aforementioned will turn into the method of "Elliptical Iterative Optimization for the convex set" (known as Elliptical Curves Cryptography or ECC) which may act as an alternative to the present RAS (Rivest-Shamir-Adleman) cryptography. It is also an application of this author's present series in the topic of the proof to Riemann Hypothesis. In a simplified version, let us consider the following procedure:

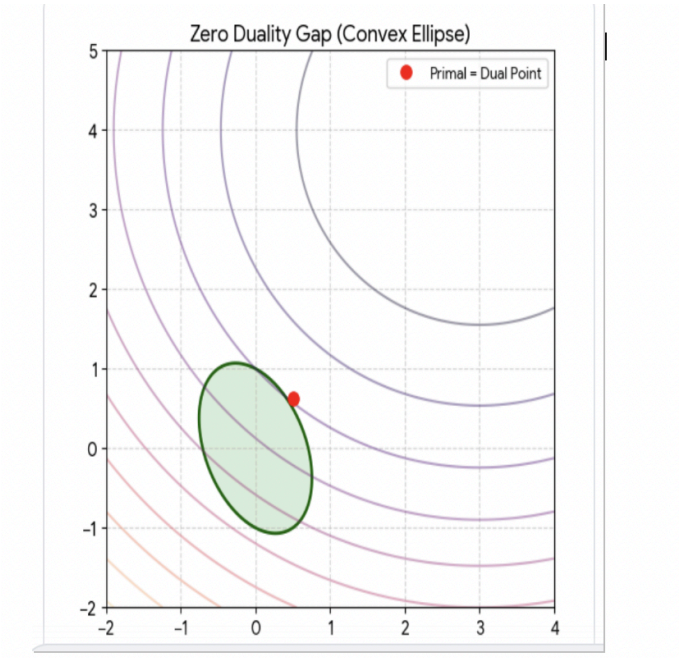
1. y2 = (Ax+1) (Bx+1) (Cx+1); or the message
2. Homogenizing the above curve gives: Y2Z = (AX+Z)(BX+Z)(CX+Z); or Z is the part 1 encryption by a person A
3. Perform the change of coordinate by first computing the L.C.M. of (A, B, C), the we may get:
   - Z = [L.C.M. (A, B, C)] Z'

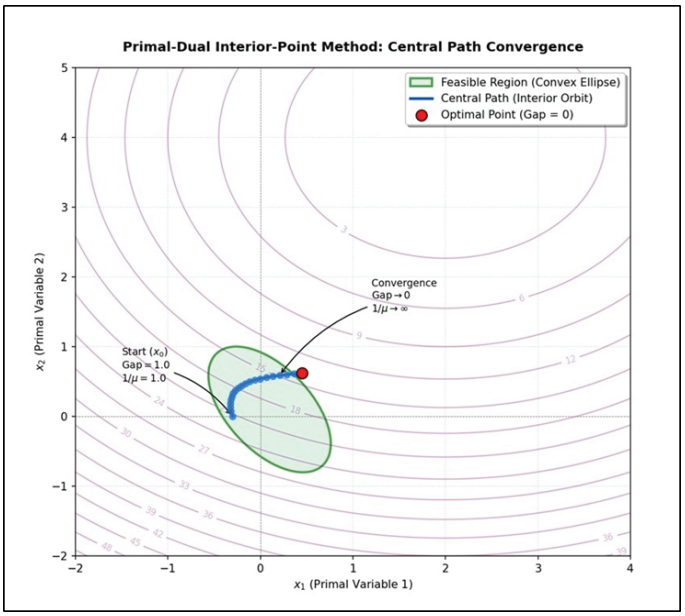
- E': y'2 = (x'+ [L.C.M. (A, B, C)]/A)(x'+[L.C.M. (A, B, C)]/B)(x'+[L.C.M. (A, B, C)]/C)
- where (x', y') = (X/Z',Y/Z') and (x, y) = (x' / [L.C.M. (A, B, C)], y' / [L.C.M. (A, B, C)]); or [L.C.M. (A, B, C)] is the part 2 encryption of a person A

4. Searching for the points in the E' which are divisible by [L.C.M. (A, B, C)]; or the part 1 decryption for a person B;
5. Input the E' equation into the mathematical software such as the SageMath and use the E.integral_points() command to compute all of coordinates that are the multiple of [L.C.M. (A, B, C)] are the expected answer; the multiply value is the unique private key (2 part) for a decryption of the message to the person B. We may convert the found coordinates back to the original equation's coordinates by a division of [L.C.M. (A, B, C)] or

answer = {[expected coordinates] / [L.C.M. (A, B, C)]}.

Actually, the aforementioned procedure or algorithm forms a set of public key and private key for encryption and decryption. Certainly, the mirror image converse of the encryption and decryption procedures are also true. In reality, if [L.C.M. (A, B, C)] tends to a zero (or an infinity), then the "answer" or the original equation will go to an infinity (or a zero). In such a case, we may thus need to apply the "primal-duality" method of the optimization just like the previous one mentioned above (*****) for a computation of the maximize the [minimum] first and then find its reciprocal – minimize the [maximum] value (or a value of "15625" for the aforementioned case, or vice versa, we may have: Minimize the [maximum] and next step is maximize the [minimum] in a given specific range. If such kind of the relationship forms a strong duality, then the duality gap is zero, i.e. where the solution of the Primal = the solution of the dual, or a perfect success in computing the wanted/expected answer to the problem.) This writer will NOT repeat. The following is a worked example showing the perfect duality gap in an elliptical problem.




```
import numpy as np
import matplotlib.pyplot as plt

================================================================
====================
================================================================
====================

theta_rot = np×radians(30)
R = np×array([[np.cos(theta_rot), -np.sin(theta_rot)],
[np.sin(theta_rot), np.cos(theta_rot)]])

def is_feasible(x, y):

pts = np.vstack([x - 0.1, y - 0.1]) pts_rot = R×T @ pts
return (pts_rot[0,:]**2 / 0.5**2 + pts_rot[1,:]**2 / 1.0**2) <= 1
 X_grid, Y_grid = np.meshgrid(np.linspace(-2, 4, 400), np.linspace(-2, 5, 400)) Z_obj = (X_grid - 2)**2 + (Y_grid - 4)**2
```

```
============================================================
====================
============================================================
====================

opt_x, opt_y = 0.45, 0.G2

start_x, start_y = -0.3, 0.0

iterations = 20
mu_history = np.logspace(0, -5, iterations)   inverse_gap_history = 1.0 / mu_history

t = np×linspace(0, 1, iterations)

path_x = start_x + (opt_x - start_x) * (t**1.5) - 0.15 * np.sin(np.pi * t) path_y = start_y + (opt_y - start_y) * (t**0.8) + 0.10 * np.sin(np.pi * t)

============================================================
====================
============================================================
====================
plt.figure(figsize=(U, 8), dpi=100)

contours = plt.contour(X_grid, Y_grid, Z_obj, levels=18, colors='purple', alpha=0.3, linewidths=1.2)
plt.clabel(contours, inline=True, fontsize=8, fmt='%1.0f')

t_ellipse = np.linspace(0, 2*np.pi, 200)
ellipse_raw = np.vstack([0.5 * np.cos(t_ellipse), 1.0 * np.sin(t_ellipse)])

ellipse_rot = R @ ellipse_raw
plt.fill(ellipse_rot[0,:] + 0.1, ellipse_rot[1,:] + 0.1, color=' d4edda', alpha=0.7, edgecolor='g', linewidth=2, label='Feasible Region (Convex El-
lipse)')

plt.plot(path_x, path_y, color=' 005Gb3', linestyle='-', linewidth=2.5, label='Central Path (Interior Orbit)')

plt.scatter(path_x[:-1], path_y[:-1], color=' 007bff', s=30, zorder=4, alpha=0.7)

plt.scatter(opt_x, opt_y, color='red', s=100, marker='o', edgecolors='black', zorder=5, label='Optimal Point (Gap = 0)')

plt.annotate('Start ($x_0$)\nGap$={:.1f}$\n$1/\\mu={:.1f}
$'.format(mu_history[0], inverse_gap_history[0]), xy=(start_x, start_y), xytext=(-1.5, 0.5), arrowprops=dict(arrowstyle="->", color='black',
connectionstyle="arc3,rad=-0.2"), fontsize=U)

plt.annotate('Convergence\nGap$\\to 0$\n$1/\\mu \\to \\infty$', xy=(path_x[-4], path_y[-4]), xytext=(1.2, 1.5), arrowprops=dict(arrowstyle="->",
color='black',
connectionstyle="arc3,rad=0.2"), fontsize=U)

plt.xlim(-2, 4)
plt.ylim(-2, 5)
plt.axhline(0, color='gray', linestyle='--', linewidth=0.5) plt.axvline(0, color='gray', linestyle='--', linewidth=0.5)
plt.title("Primal-Dual Interior-Point Method: Central Path Convergence", fontsize=12, fontweight='bold', pad=15)
plt.xlabel("$x_1$ (Primal Variable 1)", fontsize=10) plt.ylabel("$x_2$ (Primal Variable 2)", fontsize=10) plt.legend(loc='upper right',
frameon=True, shadow=True) plt.grid(True, linestyle=':', alpha=0.5)

plt.show()

============================================================
====================
============================================================
====================

print(“GenStep (k) | PrimDuGap (mu) | InvPrimDu (1/mu) -> BarrParam t") print("-" * 55)
for k in [0, 4, U, 14, iterations-1]:
print(f" Iter {k:2d}  | {mu_history[k]:.Gf}              |
{inverse_gap_history[k]:12.2f}")
```

```
GenStep (k) | PrimDuGap (mu) | InvPriDual (1/mu) -> BarrParam t
-----------------------------------------------------------
  Iter  0     |   1.000000     |            1.00
  Iter  4     |   0.088587     |           11.29
  Iter  9     |   0.004281     |          233.57
  Iter 14     |   0.000207     |         4832.93
  Iter 19     |   0.000010     |       100000.00
```

Based on the provided documents and optimization logs, the route of the primal-dual interior-point method toward a zero duality gap follows a structured geometric path (cite: 2, 5):

### 1. The Starting Position (Iter 0)

- **Initial State**: The process originates deep inside the elliptical feasible region at the starting point $x_0$ = (-0.3, 0.0) (cite: 2, 3).
- **Gap Condition**: At this stage, the primal-dual gap (μ) is wide at 1.0, meaning the distance between the upper bound (primal objective) and lower bound (dual objective) is at its maximum (cite: 2, 5).
- **Inverse Value**: The inverse primal-dual gap (1/μ) begins at 1.0 (cite: 2, 5).

### 2. The Central Path Trajectory (Iters 4 to 14)

- **Interior Orbit**: Rather than jumping directly to the boundary, the solver charts a smooth, curved path known as the **Central Path** (cite: 2, 4).
- **The Logarithmic Barrier**: A penalty parameter controls this trajectory, balancing the reduction of the true objective function with a force that keeps the path from crashing prematurely into the constraint walls (cite: 4).
- **Rapid Gap Compression**: As the algorithm steps forward, the duality gap μ shrinks exponentially (e.g., dropping sharply to 0.088 by Iteration 4, and 0.0002 by Iteration 14) (cite: 5). Simultaneously, the inverse gap (1/μ) rapidly blows up, signaling that the dual and primal objective planes are tightening toward each other (cite: 2, 5).

### 3. Convergence to the Optimum (Iter 19)

- **Final Boundary Destination**: The central path terminates exactly on the northeast edge of the convex ellipse at the coordinates (0.45, 0.62) (cite: 2, 3).
- **Perfect Duality Achieved**: At this final point, the gap μ approaches zero ($\mu \to 0$), meaning strong duality has taken effect (cite: 2, 5). The primal minimum energy balances perfectly with the dual maximum energy (cite: 2, 5).
- **Infinite Inverse Limit**: Because the actual gap vanishes, the inverse gap spikes toward infinity ($1/\mu \to \infty$) (cite: 2, 5). This confirms that the primal and dual points have mathematically fused into a single optimal solution (cite: 2).

Or similarly, we may have the Fourier expansions for the invariants g2 and g3 in the Weierstrass P function by computing the corresponding discriminant and Klein's modular function (may be further explored to find the corresponding Fourier series and hence the filter function (but NOT for filtering the social & political counterparts) as well as the digital electronic signal processing device (may be a control system with filter and feedback). At the same time, the coefficients of the such Fourier expansion c(n) have a number of interesting modular division or arithmetic/congruences properties. Certainly, it is no doubt that

the mirror image inverse (or the vice versa) way can also be true under the application of the Inverse Fourier Transform through the computer programming software to restore back into the original data as a kind of ECC decryption method etc. Thus, these properties may also be applied in another form of the Elliptic Curves Cryptography as in the cases of RSA or the present ECC one. In practice, ECC may be more secure than RSA as there may be an optimum amount of security with relative short key and imply a less computing power or network load etc. A last word is for those readers who may feel interested with the number theory, elliptical curve and the cryptography, one may refer to the book as shown in the references. Certainly, we may finally have the modular elliptic curves with the proof to the Fermat's last theorem as the we may establish the necessary and essential algebraic groups, rings and fields for solving the problem. But this is out of the scope of the paper's discussion – the Riemann Hypothesis, this author will leave such topic to those interested parties or else this author's future work if the conditions such as time is available.)

It is also true for my self-developed HKLam statistical model theory as the theory has been verified by both of the qualitative proof in pure mathematics and quantitatively by the computational data (or applied mathematics) shown in the previous section's analogy for the description of an experiment in the University of Hong Kong dental science research.

## Conclusion

In a nutshell, this author conclude that we may extend the truth of the proof to the Riemann by both of the Laurent Series together with the Dirichlet Eta function for the case of $0 < s < 1$. In addition, we may also optimize both of the Riemann Zeta Function's root of empirical model equations. Lastly, when we are optimizing the Schoenfeld inequality, we may finally approach to the relationship between the elliptical curves and the elliptical functions. When one is going forward for a step, one may get into the issue of elliptical curves, number theory and the cryptography for a more in depth and relative fully discussion. Last but not least, this writer wants to note that in reality, there may be Not-ch Filter for us to filter those zeros and also the Band filter for filtering infinity. Moreover, one may use either python (for offline applications) or Javascript (web audio applications) for the simulations to both of the above two filters. In practice, both of the encryption and decryption (or actually either finding zeros or infinities) method as described in the aforementioned paragraphs may be obtained by a software way through python or javascript programming segment like the Butterworth filter design etc.

## References


1. Lam Kai Shun (2023) A Full and Detailed Proof for the Riemann Hypothesis & the Simple Inductive proof of Goldbach's Conjecture, International Journal of Mathematics and Statistics Studies, Vol.11, No.3, pp.1-10 https://eajournals.org/ijmss/wp-content/uploads/sites/71/2023/10/A-Full-and-Detailed-Proof.pdf
2. K.S.Lam (2024) A Quantization for the Toy Model of Black Hole and a Suggestion to the Unification Theory, International Journal of Mathematics and Statistics Studies, volume 12, issue 4, pages 62-93 https://eajournals.org/ijmss/wp-content/uploads/sites/71/2024/07/A-Quantization-for-the-Toy-Model.pdf
3. Radovanović et al., (2021) The (theta, wheel)-free graphs Part IV: Induced paths and cycles, Journal of Combinatorial Theory, Series B. https://hal.science/hal-03060185/document
4. Pichmony Anhaouy,2003, PhD thesis proposal, University of British Columbia, Institute of Applied Mathematics. www.iam.ubc.ca/~pichmony/thesispr.pdf
5. Radovanović, Trotignon & Vušković, 2021 The (theta, wheel)-free graphs Part IV: Induced paths and cycles, Journal of Combinatorial Theory, Series B. eprints.whiterose.ac.uk/161693/1/twf4-revised.pdf
6. Ryerson and authored by Wayne Erdman et al. 2008, Advanced Functions 12, McGraw-Hill, ISBN: 978-0070266360. idoc.pub/documents/mhr-advanced-functions-12-546g9ojok7n8
7. Larson, R. (2007). Precalculus: A graphing approach (5th ed.). Brooks Cole. silo.pub/precalculus-a-graphing-approach-5th-edition.html
8. Plan de Adquisiciones PMSJ 2011, Peru's Proyecto de Modernización del Sector Justicia (PMSJ) pmsj-peru.org/wp-content/uploads/2012/03/Plan-de-Adquisiciones-PMSJ-2011.pdf
9. French Preparatory Classes (Prepa Commerciale) B/L section, Banque d'Épreuves Littéraires. www.courscapitole.com/lib/exe/fetch.php/niveau/prepa/commerciale/ressources/bl2004.pdf
10. T.O. Alio and E.N. March 2022, Infrastructure for Teaching and Learning in Technical Colleges in Delta State, International Journal of Science Academic Research, Volume 3, Issue 3, pages 3531–3535. www.scienceijsar.com/sites/default/files/article-pdf/IJSAR-3532..pdf
11. Ralf Sattler, 2009 diplomarbeit , Humboldt-Universität zu Berlin is titled Analytische und algebraische Aspekte von Yang-Mills-Theorien, Supervised by Prof. Dr. Jan Plefka and Prof. Dr. Jochen Brüning. qft.physik.hu-berlin.de/wp-content/uploads/2012/10/DA-ralf.pdf
12. K3 report , Stockholm University Department of Mathematics , project document for studies in mathematical statistics or actuarial science . kurser.math.su.se/pluginfile.php/16103/mod_folder/content/0/2022/2022_K3_report.pdf?forcedownload=1
13. Varona Malumbres, J. L. (2018). Recorridos por la Teoría de Números (2nd ed.). Ediciones Electolibris / Real Sociedad Matemática Española (RSME). www.unirioja.es/cu/jvarona/downloads/teoria_numeros_2aed_ipad.pdf
14. Dansk Lokomotivmands Forening. (1980, February 10). Dansk Lokomotivtidende, 80(2). jernbanearkivalier.dk. www.jernbanearkivalier.dk/tidsskrifter/dansk-lokomotivtidende/1980/Dansk%20Lokomotivtidende-Nr-2-10-Februar-1980.pdf
15. Tosio Kato, 1994, The Navier-Stokes Equation for an Incompressible Fluid in R2 with a Measure as the Initial Vorticity., Indiana University Mathematics Journal, Volume 43, Issue 3, pages 937–950. www.iumj.indiana.edu/iumj/PDF/kato972.pdf
16. Moypemna Sembona, C. C. (2016). Caractérisations des modèles multivariés de stables-Tweedie multiples [Doctoral dissertation, Université de Franche-Comté]. HAL Thèses. tel.archives-ouvertes.fr/tel-01661524/file/these_A_MOYPEMNA-SEMBONA_Clovis_2016.pdf
17. Vargas Cárdenas, 2013, master's thesis, designing a health and safety management system for "Transportes y Logística de Colombia S.A.S, Universidad Industrial de Santander. tangara.uis.edu.co/biblioweb/tesis/2013/147411.pdf
18. A.M. Al-Sadi 2023, The Impact of Artificial Intelligence in Healthcare: A Comprehensive Review, Japan Journal of Research www.sciencexcel.com/articles/sXlU6yrMQtJIcoq4si0D1G2spnlOLfbxBGTaLiCU.pdf
19. Mustapha Laayouni, 2014, Sous-algèbre commutative définie dans l , HAL open access archive ,hal.science/hal-00473768v2/document
20. Goldwasser, S., & Bellare, M. (2008). Lecture notes on cryptography. cseclassic.ucsd.edu/~mihir/papers/gb.pdf
21. Rapatskaya, L. A. (2014). Preodolenie "dukhovnoy netselnosti" kak put razvitiya kulturologicheskogo napravleniya v rossiyskoy pedagogike muzykalnogo [Overcoming "spiritual incompleteness" as a way of developing the culturological direction in Russian music pedagogy].

Chelovek i obrazovanie, (2), 15–20.cyberleninka.org/article/n/293498
22. OEIS Foundation Inc. (2024). Integers whose prime factorization consists of primes with exponent 2. Entry A249630 in The On-Line Encyclopedia of Integer Sequences. oeis.org oeis4.oeis.org/A249630
23. Goldwasser, S., & Bellare, M. (2008). Lecture notes on cryptography. utsa.edu [1, 2] www.cs.utsa.edu/~wagner/CS4363/resources/goldwasser/gb.pdf
24. Gosstandart USSR. (1986). GOST 7921-86: Metal shot-gun cartridge-cases. Specifications. Moscow: Standards Publishing House. gostrf.com/norma_data/7/7921/index.htm
25. Goldwasser, S., & Bellare, M. (2008). Lecture notes on cryptography. Brown University Computer Science Department. brown.edu cs.brown.edu/courses/cs151/reference/goldwasser_bellare_notes.pdf
26. Boyer, M., Lasserre, P., Mariotti, T., & Moreaux, M. (2004). Preemption and rent dissipation under price competition. International Journal of Industrial Organization, 22(3), 309–328. doi.org coek.info/pdf-preemption-and-rent-dissipation-under-price-competition-.html
27. Józsa, L., & Lipták-Galgóczi, Z. (2011). Felsőoktatási minőségjavítás projektmenedzsmenttel [Improving the quality of higher education through project management]. Felsőoktatási Szemle, 54(3), 205–216. publikacio.uni-eszterhazy.hu/3015/
28. Escobar Macualo, F. H. (2012). Fundamentos de ingeniería de yacimientos (2a ed.). Neiva, Colombia: Universidad Surcolombiana. scribd.com pt.scribd.com/doc/142717548/Fundamento-de-Ingenieria-de-Yacimientos-1-1
29. A convex optimization method to solve a filter design problem https://www.sciencedirect.com/science/article/pii/S0377042713002501
30. Convex Optimization of Nonlinear Feedback Controllers via Occupation Measures https://groups.csail.mit.edu/robotics-center/public_papers/Majumdar14.pdf
31. Geometric Solution to Riemann Hypothesis https://www.youtube.com/watch?v=XspbmnlQOZY
32. Jonathan Leake – Log-concave polynomials, lattice point counting and traveling sale-person problem
33. Shun L.K. (2021). AN ALGORITHMIC APPROACH TO SOLVE CONTINUUM HYPOTHESIS. International Journal of Mathematics and Statistics Studies. Vol.9, No.2, pp.69-87
34. Carson L.K.S. (2023) Predictive and Regenerative Medicine for Humans, European Journal of Biology and Medical Science Research, Vol.11, No.4, pp.,27-39
35. Michael Penn (2024). Defense against the “dark art” of mathematics. https://www.youtube.com/watch?v=WCthfLpYA5g
36. Shun L.K. (2021) INTEGRATED CIRCUITS: SYMMETRIC USAGE IN REVERSING PARALYSIS, European Journal of Computer Science and Information Technology, Vol.9, No.3, pp.8-18
37. Shun L.K. (2023) An Innovate Comparative Research Method to determine the convergence (/or divergence) of the Infective Virus like COVID-19 & Common Flu, International Journal of English Language Teaching, Vol.11, No.4, pp.,7-19
38. Physics with Elliot (2023). To understand Fourier Transform, Start from Quantum Mechanics. https://www.youtube.com/watch?v=W8QZ-yxebFA
39. Shun L.K. (2022) Charging Our Electrical Devices in Anywhere and at Any time, International Journal of Mathematics and Statistics Studies, Vol.10, No.5, pp.70-89
40. Overholt, M. (2014). A course in analytic number theory. American Mathematical Society. https://www.youtube.com/watch?v=kvvpGW_dD1E
41. The Holy Trinity of curves, Prof Michael Penn https://math.stackexchange.com/questions/4372860/defining-an-elliptic-curve-by-a-cubic-equation
42. Abbott, S. (1991). Modular functions and Dirichlet series in number theory, 2nd edition, by T. M. Apostol. Pp 204. DM98. 1990. ISBN 3-540-97127-0 (Springer). Mathematical Gazette, 75(472), 249–252. https://doi.org/10.2307/3620307
43. https://www.quora.com/Why-are-we-allowed-to-extend-the-Riemann-zeta-function-to-numbers-smaller-than-equal-to-one
44. Washington, L. C. (2008). Elliptic curves : number theory and cryptography (2nd ed.). Chapman & Hall/CRC. https://doi.org/10.1201/9781420071474