

A Fundamental Problem in Physics - (Mass vs Electric Charge)

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Abstract

In modern physics there exist opposite concepts regarding the constancy and the variability of two major properties of the electron: its mass and its electric charge.

Section 1: Mainstream physicists consider the mass of the electron as varying with its velocity while its electric charge as a fundamental constant of physics.

Section 2: On the other hand, many other physicists maintain the opposite concept: the mass of the electron is invariant in all physical conditions while its electric charge is variable.

Section 3: In the case when the mass is invariant, a mathematical expression for the variability of the electric charge of the electron in external fields will be deduced.

Section 4: A thought experiment to demonstrate the variability of the electric charge of the electron in a changing magnetic field.

Section 5: Discussions

- i) Why is the electric charge changed in external field?
- ii) Renormalizing the mass is problematic
- iii) Renormalizing the electric charge is innovative
- iv) The mystery of the mass of the muon
- v) The controversial concept of time dilation

Mainstream physics: the mass of an elementary particle (e.g. an electron) varies with its velocity

This idea came from the theory of the electron of Lorentz in which he proposed (1904) an extended model of the electron as a uniform spherical surface charge. When this electron moved through the “ether”, its transverse dimension remained unchanged, but its length in the motion direction was contracted and the variation of mass with velocity was derived as

$$m = (1 - v^2/c^2)^{-1/2} m_0 \quad (1)$$

We can also find various expressions which describe the variation of the mass with velocity in current textbooks. For example, in the textbook “Introduction to the Theory of Relativity” by Bergmann [1] (1976) we can read in the topic “Relativistic Force”:

“In general, the force thus defined is not parallel to the acceleration. It is parallel only when the acceleration is either parallel or perpendicular to the velocity

When it is parallel, it takes the form

$$f = (1 - u^2 / c^2)^{-3/2} m a \quad (2)$$

When the force and velocity are orthogonal, it becomes

$$f = (1 - u^2 / c^2)^{-1/2} m a \quad (3)$$

The coefficients of the acceleration on the

right-hand sides of these two equations are occasionally referred to as “longitudinal mass” and “transversal mass”, respectively”.

So, from (2) the longitudinal mass is

$$m_l = (1 - u^2/c^2)^{-3/2} m = \gamma^3 m, \text{ when } \mathbf{a} // \mathbf{v} \quad (4)$$

and from (3) the transversal mass

$$m_t = (1 - u^2/c^2)^{-1/2} m = \gamma m, \text{ when } \mathbf{a} \perp \mathbf{v} \quad (5)$$

where the velocity is denoted by \mathbf{u} or \mathbf{v} and $\gamma = (1 - v^2 / c^2)^{-1/2}$ is the Lorentz factor; m_l and m_t thus depend on the velocity and also on the direction of motion of the particle relative to the external field.

We can also find these two expressions (4) and (5) of the longitudinal and transversal masses in the textbook “Classical Dynamics of Particles and Systems” by Marion and Thornton [2] (1995), p.576.

The opposite concept: the mass of an elementary particle is always constant.

Like in politics, physicists always confront with the opposition! The opposite concept is that the mass of a particle is always constant in all physical conditions.

Let's read the following quotations to see how other physicists confirm the invariability of the mass of a particle.

- i) **Okun** [3], "The concept of mass", Physics Today, 1989 “In the modern language

of relativity theory there is one mass, the Newton mass m , which does not vary with velocity”.

- ii) **Sternheim & Kane** [4], "General Physics", 1991 “ *The correct definition of the relativistic momentum of an object of mass m and velocity v is $p = mv (1 - v^2/c^2)^{-1/2}$. In this equation, m is the ordinary mass of the object as measured by an observer in its rest frame. (Some books refer to this quantity as the rest mass and also define a velocity-dependent mass. We do not do this)*”.
- iii) **Marion & Thornton** [2], "Classical Dynamics of Particles and Systems", 1995, p.555 “*Scientists spoke of the mass increasing at high speeds. We prefer to keep the concept of mass as an invariant, intrinsic property of an object. The use of two terms relativistic and rest mass is now considered old-fashioned. We therefore always refer to the mass m , which is the same as the rest mass*”.
- iv) **Kacsner** [5], "Encyclopedia of Physics", by Lerner & Trigg, 2005, topic: “Relativity, Special Theory” “ *Mass – a notational issue - yet profoundly important. In many relativity presentations (but generally not in Einstein’s own works), a misleading set of mass definitions was created – rest mass, relativistic mass (an abomination), transverse mass, etc. It has been strongly and correctly argued by Okun that these confusions should not be propagated. So here I will use m as the one-and-only mass of a particle being what is often called the rest mass and written m_0 . This mass m (by others often called m_0 or the rest mass) is the same as the Newtonian mass at low velocities. Most important, m is a scalar or invariant, it has the same value for all observers of the particle, and is a constant parameter for the particle. It is to be determined by experiment, and by use of relativistic dynamics*”.
- v) **Adler** [6], Am. J. Phys. 55, (1987); “Does mass really depend on velocity, dad?” In the letter to Lincoln Barnett, 19 June 1948, **Einstein** wrote (in German): “*It is not good to introduce the concept of mass $M = m / (1 - v^2/c^2)^{1/2}$ of a moving body for which no clear definition can be given. It is better to introduce no other mass concept than ‘the rest mass’ m . Instead of introducing M , it is better to mention the expression for the momentum and energy of a body in motion.*”

Conclusion: From these quotations we come to the conclusion that in the contemporary physics there exists an obvious conflict in the concept of mass about whether or not it is invariant.

This is a fundamental problem in physics and a plausible solution is proposed in the following sections.

To do this, first we notice that the measurements of the value of the ratio e/m of the electron show that **"its value decreases (owing to increase in mass) as the velocity of the electron approaches that of light"**. (Illingworth, V., The Penguin Dictionary of Physics, 2nd Ed. 1991): (e/m)

Now, if the mass of the electron is regarded as invariant (as stated by physicists in section 2), it is reasonable to say that the decrease of the ratio e / m is caused by the decrease of the electric charge e with its velocity, but not by the increase in mass.

Therefore, we can replace the concept of varying mass by the concept of varying charge in phenomena involving the motion of the electron in external field.

Search for a mathematical expression that describes the variability of the electric charge of the electron in external field

We notice in the equations of motion of the electrons in the external field that e and m appear together in the ratio e/m , rather than emerging separately. Now if we want to adjust these equations of motion to express the effect of relativistic mass of the electron, we write this ratio in the form $e/(\gamma m)$. Since $e/(\gamma m) \equiv (\gamma^{-1}) e/m$, this means that *the equation of motion does not change mathematically* if we write this ratio in either form: $e/(\gamma m)$ or $(\gamma^{-1}) e/m$.

If we write the ratio in the second form $(\gamma^{-1}) e/m$, this *physically* means that we choose to express the concept of varying charge $(\gamma^{-1}e)$ instead of the concept of varying mass (γm) .

We notice that the renormalizing factor for the electric charge (γ^{-1}) is the inverse of the renormalizing factor for the mass (γ) .

This means that if γ is known, we can deduce the inverse γ^{-1} .

Below we will use this statement to search for a mathematical equation which describes the variability of the electric charge of the electron in external fields.

We have seen in section 1: the renormalizing factor γ for the mass has at least two different exponents: 3 and 1

- γ^3 for the longitudinal mass: $m_l = \gamma^3 m$, when $\mathbf{a} // \mathbf{v}$, from Eq (4)
- γ for the transversal mass: $m_t = \gamma m$, when $\mathbf{a} \perp \mathbf{v}$, from Eq (5)

And thus, the renormalizing factors for the electric charge in these two particular cases are γ^{-3} and γ^{-1} (these are the inverses of γ^3 and γ respectively). So, we can write:

$$q = \gamma^{-3} q_0, \text{ when } \mathbf{a} // \mathbf{v} \quad (6)$$

and

$$q = \gamma^{-1} q_0, \text{ when } \mathbf{a} \perp \mathbf{v} \quad (7)$$

Two exponents 3 and 1 in Eqs (4) and (5) and in Eqs (6) and (7) originate from *two particular cases* of direction between the velocity \mathbf{v} and the acceleration \mathbf{a} : $\mathbf{a} // \mathbf{v}$ and $\mathbf{a} \perp \mathbf{v}$. Now, the

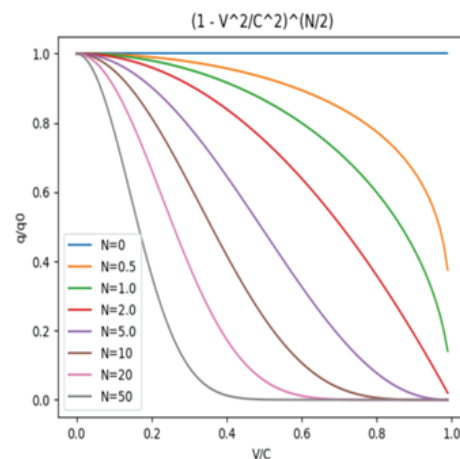


Figure 1. $q/q_0 = (1 - v^2/c^2)^{N/2}$

heuristic reasoning allows us to generalize two equations (6) and (7), for any relative direction between \mathbf{v} and \mathbf{a} in any external field by replacing these two exponents 3 and 1 by a unique real positive number N to unify two equations (6) and (7):

$$q = \gamma^{-N} q_0 = (1 - v^2/c^2)^{N/2} q_0 \quad N \geq 0 \quad (8)$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$ is the Lorentz factor. Figure.1: the graph of Eq(8) shows the *magnitude* of the electric charge of the electron in function of its velocity and the applied field which is represented by the real number N .

From the graph we notice that the higher the velocity and/or the stronger the applied field (N), the lower the effective charge q of the electron. This means that when the electron is subject to an external applied field, its effective electric charge drops below its original charge q_0 (which is conventionally denoted as e).

Some remarks:

- (i) **At low velocity:** $v \ll c$, $q \approx q_0$ for all values of N ; i.e., for all applied fields. This is the case of **Millikan** [7]. Oil-droplet experiment. In this experiment, electrons (on oil droplets) fall down at low velocity of a **fraction of a millimeter per second in the electric field of 6000 volts per cm**. And as a result, Millikan experiment could only give the unique value $q \approx q_0 (\equiv e)$ for the electric charge of the electron. Mainstream physicists considered this value e as the only value for the electric charge because after Millikan, they could not perform this experiment at higher velocities or in stronger fields which might give the effective charge q other values different from e^*
- (ii) **At high velocity near c** ($v \rightarrow c$), q depends on the intensity of the applied field N :
 - if $N = 0$ (in free space): $q = q_0$ for all velocities.
 - if $N = 0.5$ or 1.0 , the charge q drops but does not reach zero when $v \rightarrow c$;
 - if $N = 2.0$, $q \rightarrow 0$ when $v \rightarrow c$;
 - if $N = 5.0, 10, 20, 50$, $q \rightarrow 0$ at velocities less than c .
- (iii) **In intense fields:** the curves with $N = 5.0, 10, 20, 50$ demonstrate that in intense fields, electrons could be devoid of their charge and become *free particles* ($q \approx 0$) at *velocities less than c* . Equation (8) is the only equation that shows the impact of the applied field on the electric charge of the electron. Thus, while the electron is accelerated in the accelerators, its electric charge changes continuously; it does not remain constant as being thought so far.
- (iv) The immediate consequence of Eq(8) is the adjustment of the Lorentz's force equations for relativistic regime:

$$\mathbf{F}_L = (1 - v^2/c^2)^{N/2} q_0 (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

in which the electric force \mathbf{F}_e and the magnetic force \mathbf{F}_m tend to zero as $v \rightarrow c$:

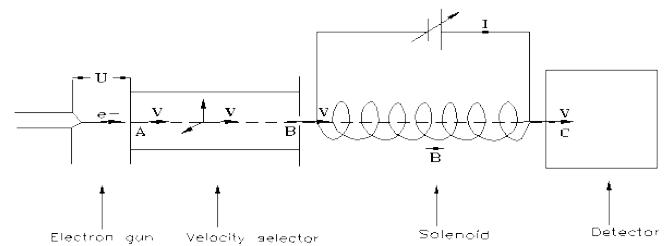
$\mathbf{F}_e = (1 - v^2/c^2)^{N/2} q_0 \mathbf{E}$: F_e decreases with v and tends to zero as $v \rightarrow c$.

$\mathbf{F}_m = (1 - v^2/c^2)^{N/2} q_0 \mathbf{v} \times \mathbf{B}$: F_m first increases with v , reaches its maximum $v = c (N + 1)^{-1/2}$, then decreases and tends to zero as $v \rightarrow c$.

For $v \ll c$, $\mathbf{F}_e \approx q_0 \mathbf{E}$ and $\mathbf{F}_m \approx q_0 \mathbf{v} \times \mathbf{B}$: these are familiar non-relativistic Lorentz's force equations.

A thought experiment to demonstrate the variability of the electric charge of the electron

If the electric charge of the electron is an effective one which varies with the applied field, we can figure out an experiment to



demonstrate this variability (Figure 2).

In this thought experiment we keep the velocity of the electrons unchanged while we change the strength of the magnetic field \mathbf{B} in the solenoid by changing the intensity of the current \mathbf{I} .

- An electron gun produces electrons with various velocities at the point A.
- A velocity selector allows only electrons with velocity v to travel to the point B.
- A solenoid produces a uniform magnetic field \mathbf{B} along its axis which coincides with the trajectory of the electron beam. The intensity \mathbf{B} of the magnetic field can be regulated by the current \mathbf{I} . Since $\mathbf{v} \parallel \mathbf{B}$, there is no net (magnetic) force produced on the electron, so electrons travel with constant velocity v through the solenoid to the point C. And thus, there is no change in the mass and the kinetic energy of the electron with velocity.
- A detector, which can be a thick block of silver bromide (photographic emulsion), is installed at the exit C of the solenoid to detect the changing of the electric charge q of the electron when \mathbf{B} changes its intensity.

At the entrance point C on the detector, the velocity of the electron is v , and its effective charge is q , which is expected to decrease when the magnetic field \mathbf{B} significantly increases.

Since the energy loss per unit distance [4] in the medium of the detector is proportional to q^2/v^2 , (that is $\Delta K \propto q^2/v^2$), if the intensity of \mathbf{B} increases (N increases), the effective electric charge q will drop (according to the curves in Figure 1) and hence Δ decreases, resulting in a deeper penetration of electrons into the block of photographic emulsion.

In short, when we change the intensity of \mathbf{B} , if the change of penetration responds to the change of \mathbf{B} , this proves that q varies with the applied magnetic field.

We expect that the stronger the magnetic pulses, the deeper the penetration becomes, and when the magnetic pulses becomes

* It is interesting to read the following remarks that Millikan made on his experiment of oil-droplets: "In order to be able to measure very accurately the force acting upon the charged oil-droplet it was necessary to give it about a centimeter of path in which the speed could be measured. This is one of the most important elements in the design, the overlooking of which has caused some subsequent observers to fall into error ... The field strength too, about 6,000 volts per cm, was vital, and new in work of anything like this kind. It was the element which turned possible failure into success. Nature here was very kind. She left only a narrow range of field strengths within which such experiments as these are all possible." (Millikan's Nobel lecture, 1924)
We notice that in his Nobel lecture, Millikan never said that the electric charge e of the electron was an invariant. So, if this experiment could be carried out at higher velocities and/or in a much stronger electric field, would it result in other values different from e ?

sufficiently intense, $q \rightarrow 0$: the interactions between electrons and molecules of silver bromide of the detector vanish, the free electrons eventually traverse the medium.

Discussions

i) Why is the electric charge of the electron changed by external fields?

We wonder why the electron changes its electric charge in external fields. The plausible answer is because it is not a rigid point charge, but an extended and structured particle, and hence its electric charge is affected by the impact of the external field [11].

ii) Renormalization of mass is problematic.

The renormalization of the mass m of the electron sends it to infinity: $m \rightarrow \infty$ as $v \rightarrow c$. To avoid the infinity, physicists have come up with the renormalizing procedure, which many other physicists called a disaster:

- **Dirac (1975):** "This so-called 'good theory' does involve neglecting infinities which appear in its equations, ignoring them in an arbitrary way".
- **Feynman (1985):** "It is still what I would call a dippy process!... I suspect that renormalization is not mathematically legitimate".
- **Ryder:** "In the Quantum Theory, these [classical] divergences do not disappear, on the contrary, they appear to get worse". (From Wikipedia: Renormalization)

iii) Renormalization of electric charge is innovative.

The ideas of charge renormalization appeared in the literature with

- **Schwinger, Bekenstein, Rohrlich, Schwinger [8]** "Now one of the most important interaction aspects of quantum electrodynamics is the phenomenon of vacuum polarization. The implication that physical charge are weaker than bare charge by a universal factor is the basis for charge renormalization".
- **Bekenstein [9]:** "Thus every particle charge can be expressed in the form $e = e_0 \in (x^{\mu})$, where e_0 is a constant characteristic of the particles and \in a dimensionless universal field".
- **Rohrlich [10]** wrote in the topic of renormalization: "The effective charge e , which is the physical (renormalized) charge, is defined to be

$$e = Z_1 Z_2^{-1} Z_3^{-1/2} e_0$$

where Z_i are renormalization constants".

In the renormalization of electric charge: $q \rightarrow 0$ as $v \rightarrow c$ and there is no infinity caused by the mass m of the electron because it is considered as invariant. The result is the Eq(8) and its graph shown in Figure 1 which can be used to explain some features of the electron: the mystery of the mass of the muon and the controversial dilation of its lifetime.

iv) The mystery of the muon: the muon is the electron with reduced electric charge.

Physicists consider the muon as a mystery:

- "Muons even today represent something of a puzzle. Only in its mass and stability does the muon differ significantly from the electron, leading to the hypothesis that the muon is merely a kind of 'heavy electron' rather than a unique entity". (A.Beiser, Concept of Modern Physics, 1981)

- "The muon is a mystery; it is like an electron almost every way but its mass. There is no known reason why it must exist. A complication is introduced by the magnitude of the charge, for both the rate of energy loss and the radius of the circle depend on how much charge the particle has". (Lehrmann & Swartz, Foundation of Physics, p.697, 1969)

The last quotation tells us that the magnitude of the charge interferes in the determination of the mass of a particle. So, if we do not know how much charge the particle has, we cannot determine its mass accurately; this is the case in the determination of the mass of the muon.

An explanation of the mystery of the muon is to apply the concept of variability of the electric charge of the electron as presented in section 3.

In the determination of the mass of the muon, physicists assumed that its charge is invariant, equal to e , while its mass varies with velocity. This assumption certainly is the reason why their calculations resulted in the heavy mass of the muon:

$$m_{\mu} = 207 m_e \quad (9)$$

This relation means that m_{μ} is the renormalizing mass of the electron, in which the renormalizing factor is $\tau = 207$

In section 3, we have come to the result that **the equation of motion of the electron remains unchanged if we renormalize the electric charge by the factor γ^{-1} instead of renormalizing the mass by the factor γ** .

Hence, instead of renormalizing its mass by γ as shown in Eq(9), let's renormalize its electric charge by τ^{-1} , we get

$$q_{\mu} = \tau^{-1} q_0 = (207)^{-1} q_0, \\ \text{where } q_0 \equiv e = 1.602 \times 10^{-19} \text{ C} \quad (10)$$

or

$$q_{\mu} = (207)^{-1} e = 7.739 \times 10^{-22} \text{ C} \quad (11)$$

Therefore, the muon is the electron with reduced electric charge equal to 7.739×10^{-22} C. The muon differs from the electron by its reduced, varying electric charge; its mass remains unchanged, equal to that of the electron.

- As for the varying electric charge: when the muon is created, the magnitude of its charge is 7.739×10^{-22} C; it increases to $e = 1.602 \times 10^{-19}$ C in $2.2 \mu\text{s}$ (this is the lifetime of the muon). The variation of the electric charge follows a curve N shown in Figure 1. Since the charge varies in a very short period of time ($2.2 \mu\text{s}$), it cannot be detected by any detector; it is taken for granted as a constant, equal to e , which is its final charge.
- As for the velocity: when the muon is created, its velocity v may be high; it slows down and eventually stops in the detector: $v \rightarrow 0$ in $2.2 \mu\text{s}$. At the end of the travel, it becomes identical to the electron: $\mu^- \rightarrow e^-$ (by following a curve N in Figure 1 which ends up at the terminal coordinates ($v = 0$, $q = q_0$) in $2.2 \mu\text{s}$).
- As for the mass of muon: it is invariant, equal to the mass m_0 of the electron through its transformation from muon to electron.

Conclusion: There are two different paths for the interpretation:

If the muon is considered as a particle that has *constant electric charge* $e = 1.602 \times 10^{-19} \text{ C}$ and its mass changing with velocity, then we will get a muon which is 207 times heavier than the electron: $m_{\mu} = 207 m_e$. This is the mainstream physics, for which all charged particles have constant charge: $\pm e$. This way leads to the idea that **muon is a heavy electron**.

But if we consider the muon as a particle that has *invariant mass* and its charge is changing with velocity and applied field, then we will get a muon which has the same mass as the electron $m_{\mu} = m_e = 0.511 \text{ Mev}/c^2$ but a reduced charge: $q_{\mu} = (207)^{-1}e = 7.739 \times 10^{-22} \text{ C}$.

This is an innovative way which helps explain the long-lived mystery of the mass of muon: **muon is not a heavy electron, but a reduced charged electron. And hence the muon is much more penetrative than the electron.**

Note: The **tau particle** τ is analogous to the muon: it is the electron with reduced electric charge: $\tau \rightarrow \pi \rightarrow \mu \rightarrow e$

v) The controversial concept of time dilation

The following phenomena are often cited in the physical literature to prove that the increase of the lifetime of the muon with its velocity is a proof that the *concept of time dilation is a real physical phenomenon*.

1. Experiments performed at CERN showed that muons at speed of $0.99 c$ were found to have an average lifetime 29 times as large as that of muons at rest.
2. The finding of muons at sea level proved that due to their high speed ($0.999 c$), their lifetime has increased from $2 \mu\text{s}$ to $30 \mu\text{s}$ such that they can travel over 9000 m (instead of 600 m) to reach the sea level.

From these data, some physicists believe that time dilation is no doubt a real physical phenomenon.

The purpose of this subsection (v) is to show that the **time dilation does not exist physically** although the lifetimes of moving muons actually increase with their velocity.

First of all, physicists think that all muons, no matter where and how they are created, must have the same lifetime (the average is $2.2 \mu\text{s}$). So with the velocity $v = 0.99 c = 0.99 \times 3 \times 10^8 \text{ m/s}$, the muon can travel only 653 m in $2.2 \mu\text{s}$. But since muons were found at sea level, i.e., they have travelled over 9000 m (from the upper atmosphere to the sea level). This means that their lifetime has increased from $2.2 \mu\text{s}$ to $30.3 \mu\text{s}$. From this increase of the lifetime of muons, physicists concluded that the *time dilation is a real physical nature of time*. Physicists have thus assimilated two different notions: the *increase of lifetime* (of the muons) and the *time dilation*. The mixing up of these two notions is superficial and hence incorrect because we can explain the increase of the lifetime of the muons by using the concept

of variability of the electric charge of the electron (described in section 3), in which **there is no place for the concept of time dilation**.

The explanation is as follows: because the muons are created in different places (in the labs on Earth or in the upper atmosphere), they start their lives at different coordinates (v, q) on the chart of Fig.1, and finally they end up at the terminal point $(v = 0, q = q_0)$ by following different curves N in Fig.1. By following different paths of transformation from muon to electron ($\mu \rightarrow e$), they spend different time to reach the terminal point: this means that they have different lifetime which has nothing to do with the nature of time.

In short, the concept of time dilation is unnecessary and redundant. Time does not dilate nor shrink: it is absolute as conceived by **Newton**. The concept of time dilation led to counter-intuitive ideas of "**length contraction**" and the so-called "**twin paradox**": both ideas are old-fashioned and superfluous for modern physics.

Conclusion: There are two different paths for the interpretation:

Physics changes and has no frontier. Eliminating an old-fashioned concept is as crucial for science as inventing a new one. This article proposes to replace the concept of varying mass by the innovating concept of varying electric charge. The replacement will help us better understand two enigmatic features of the electron: its spin and radiation [11].

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