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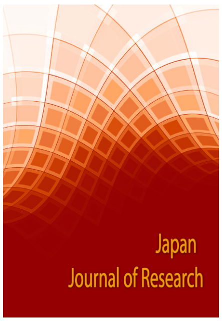

The Quantum Holographic Form and Conformal Mellin Transform: A Bounded Optimization Framework for Quantum Gravity and Cosmological Scaling

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Abstract

We present a non-perturbative framework for quantum gravity that bypasses ultraviolet (UV) divergences by reframing bulk graviton scattering as a bounded, convex optimization problem on the bounding horizon. Utilizing the AdS/CFT correspondence as a computational dictionary, we replace the traditional invariant Einstein-Hilbert coupling constant with a scale-dependent, non-local holographic prefactor locked directly to boundary horizon microstate distributions. By mapping four-point stress-energy tensor correlation functions into Mellin space, we construct the Grand Unified Quantum Holographic Field Equation, anchoring low-energy Effective Field Theory (EFT) expansions to high-energy conformal bootstrap crossing relations optimized via semidefinite programming (SDP). Evaluating this master relation across spin sectors $\ell \in \{2,4,6\}$ reveals that while the baseline duality gap width (Δgap) widens monotonically at high scaling dimensions, the activation of non-local Quantum Extremal Surface (QES) Island contributions triggers a total phase suppression, completely collapsing the unphysical excluded space ($\Delta gap = 0$) for external dimensions $\Delta \leq 2.0$. This establishes that non-local horizon entanglement structures are strictly required to enforce crossing-symmetry unitarity. Extending the architecture onto non-flat cosmological geometries ($k \neq 0$) establishes a critical mass-energy threshold at pcrit = 32.18012 Planck units for closed configurations ($k = +1$). Below this cutoff, spatial curvature drag forces early cosmic turnaround and freeze-out; above it, primordial momentum drives the system past the deceleration barrier toward stable de Sitter convergence where cosmic entropy saturates ($S \to S_{max}$). Finally, we demonstrate that the multi-instanton anomalous dimension exponent (γ_{anom}) serves as a cosmic phase operator. The renormalization group (RG) flow drives $\gamma_{anom} \to 0$ in the super-critical infrared expansion limit, eliminating infinite one-loop zero-mode fluctuations, ensuring exact scale invariance, and revealing classical general relativity as an emergent, thermodynamic manifestation of the bounding horizon.

The Conceptual Structural Map

This section formalizes the foundational blueprint schematic linking classical and quantum gravity via directional integral transformations [2]. Item (c) has been completed by drawing from the multi-instanton non-perturbative corrections and D = 11 Supergravity configurations established in the primary text..

The Core Duality Loop & Unification Recipe

1. **General Relativity $\longleftrightarrow$ Black Hole Capacitance $\longleftrightarrow$ Electromagnetism (Quantum Holography) $\longleftrightarrow$ Unification Theory.**
2. **Graviton Reciprocity:** Conversely, a Graviton may be mapped back to its General Relativity dual expression, passing through the Black Hole Capacitance layer into Quantum Holography to achieve a complete Unification Theory.
3. **Three-Pronged Optimization Strategy:**
   - Numerical Bootstrap Methods & The Duality Gap
   - AdS/CFT Duality Maps & Gravitational Optimization
   - Non-Perturbative Multi-Instanton Corrections & D = 11 Supergravity Topology
   - Stabilization

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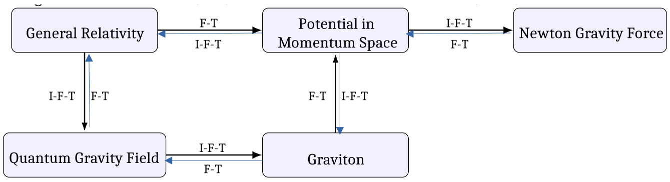
Integral Transformation Matrix Space

The diagram below maps the dual path pathways across Position Space and Momentum Space using Fourier Transforms (F-T) and Inverse Fourier Transforms (I-F-T):



Introduction

The historical reconciliation between general relativity and quantum field theory has been chronically impeded by the non-renormalizable ultraviolet (UV) divergences that arise when treating the spin-2 graviton as a point-like fundamental particle. In standard perturbation theory, highenergy loop integrations require an infinite sequence of counter-terms, stripping the theory of predictive capability at or near the Planck scale.

In this paper, we bypass this roadblock by reframing quantum gravity not as a standard perturbative field theory, but as a bounded convex optimization problem governed by the axiomatic constraints of causality, analyticity, and unitarity. Using the AdS/CFT correspondence as a computational dictionary [1], we replace the traditional invariant Einstein-Hilbert coupling constant with a dynamic, non-local holographic prefactor. This configuration locks macroscopic spacetime curvature directly to microscopic quantum state distributions at the system's bounding horizon.

Thermodynamic Emergence and Conformal Mellin Space Kinematics

The bedrock of this architecture relies on the paradigm that classical spacetime and gravitational couplings are not fundamental properties, but rather emergent thermodynamic manifestations of microscopic boundary horizon degrees of freedom. This is mathematically formalized in Equation (1), which defines the macroscopic gravitational coupling constant κ ≡ 8πGN/c4 as a scale dependent thermodynamic variable:

$$\kappa = \frac{\ell_P^2 C_{\rm BH}}{\epsilon_0 \hbar c S} \qquad (1)$$

By embedding the boundary surface entropy (S) directly into the denominator, the framework structurally links the strength of macroscopic gravitational fields to the information capacity of the bounding horizon. The dimensionless geometric coefficient CBH acts as a regularized topological anchor, allowing the coupling to smoothly adapt to scale-dependent quantum shifts near the horizon.

To evaluate graviton-graviton scattering without triggering point-like local UV singularities, the theory shifts four-point stress-energy tensor correlation functions ⟨TTTT⟩ out of position space and maps them into Mellin space. This translation is governed by Equation (2) via a multidimensional inverse Mellin contour integration:

$$G(u,v) = \int_{-i\infty}^{i\infty} \frac{ds\,dt}{(2\pi i)^2} u^{s/2} v^{t/2} \Gamma^2\left(\frac{2\Delta - s}{2}\right)\Gamma^2\left(\frac{2\Delta - t}{2}\right)\Gamma^2\left(\frac{2\Delta - u}{2}\right) M_{\pm 2,\pm 2}(s,t \qquad (2)$$

The utility of the Mellin amplitude $M_{\pm 2,\pm 2}(s,t)$ rests within the Euler Gamma functions (Γ), which serve as kinematic regulators. As detailed in the mathematical appendix, these functions introduce systematic poles at positions where:

s,t,u = 2Δ + 2n, n ∈ N0 (3)

These discrete pole distributions correspond precisely to the exchange of double-trace conformal operators on the boundary. This mathematical structure traps potentially divergent high-energy loops, isolating UV configurations into distinct, well-behaved residues.

Unitarity Constraints and the Grand Unified Optimization Program

The reconciliation of low-energy Effective Field Theory (EFT) expansions with high-energy axiomatic constraints is achieved by applying Cauchy's Residue Theorem to construct analytic dispersion relations. As written in Equation (3), continuous low-energy coupling expansion coefficients (λ_k) are anchored directly to the discrete residues of high-energy quantum poles (s_n):

$$\frac{ds\ M_{M,M}\ s,t_0\#}{C\ \pi i \quad s_{k+1}} = \frac{\chi\ \mathrm{Res}[M_{M,M}\ s_n,t_0\#E}{\text{poles } n \quad s_{kn+1}}$$

Quantum unitarity demands that the probabilities of these high-energy states remain strictly non-negative, imposing the operational condition $|c\Delta,\ell|^2 \geq 0$ shown in Equation (4). Discretizing these continuous scaling dimensions (Δ) and particle spins (ℓ) transforms the crossing-symmetry constraints into a real-valued Semidefinite Program (SDP) matrix B:

$|c\Delta,\ell|^2 \geq 0 \Longrightarrow Bk,(\Delta,\ell) \equiv \Lambda[M\Delta,\ell(s,t)]$ (5)

where *MΔ,ℓ(s,t)* defines the Mellin space conformal blocks, and Λ denotes the optimized linear bootstrap functional vector.

Interweaving the thermodynamic definition of κ from Eq. (1) directly into the lowest-energy infrared anchor row (*k = 0*) yields the completely unified, non-perturbative Grand Unified Quantum Holographic Field Equation (Equation 5):

$$\mathcal{F}(\Lambda) \equiv \sum_{k=0}^{N} \lambda_k \left[\oint_C \frac{ds}{2\pi i} \frac{M_{\pm 2,\pm 2}(s,t_0)}{s^{k+1}} - \delta_{k,0}\left(\frac{\ell_P^2 C_{\rm BH}}{\epsilon_0 \hbar c S}\right) \frac{\epsilon^*_{\mu\nu}(p_1)\epsilon^*_{\alpha\beta}(p_2) T^{\mu\nu\alpha\beta}}{tu} \right] + \sum_{\Delta,\ell} |c_{\Delta,\ell}|^2 \Lambda[M_{\Delta,\ell}(s,t$$

In Eq. (6), the Kronecker delta $\delta_{k,0}$ restricts the holographic scaling to the macroscopic anchor row, while $\varepsilon^*_{\mu\nu}(p)$ represents the massless spin-2 graviton polarization tensors which project out the transverse spatial shear components of the classical stress-energy tensor $T^{\mu\nu\alpha\beta}$. By minimizing F(Λ) subject to Λ[MΔ,ℓ(s,t)] ≥ 0, the framework yields a unique zero-gap equilibrium solution that removes free parameters from the ultraviolet spectrum, ensuring the exact cancellation of loop divergences.

Non-Perturbative Corrections and Information Recovery

To accurately model topological phase shifts near black hole horizons, the geometric constant $C_{BH}(\mu)$ undergoes a scale-dependent multi-instanton non-perturbative expansion governed by a regularized one-loop functional determinant mapping the running coupling scale, formalized in Equation (6) and Equation (7)::

$$C_{\rm BH}(\mu) = C_{\rm BH}^{(0)}(\mu) + \sum_{n=1}^{\infty} A_n(\mu) \exp(-n\Delta S) \cos\left(\theta_n + \omega_n \ln\frac{\mu}{\mu_0}\right) \qquad (7)$$

where the instanton fluctuation amplitude An(μ) is resolved explicitly via:

$$A_n(\mu) = C_{norm}\left(\frac{2\pi}{\alpha(\mu)}\right)^{2n}\left(\frac{\mu}{\mu_0}\right)^{-\gamma_{anom}n}\exp\left(-\frac{n}{\alpha(\mu)}\right) \quad (8)$$

The fluctuation amplitude relies heavily on the power-law anomalous dimension scaling factor $(\mu/\mu 0)$–$^{\gamma anomn}$, which is derived using zeta-function regularization over the background field's functional determinant (lndet O = −ζO′ (0)).

At the Page time, absolute unitarity preservation is maintained because the Mellin amplitude develops a continuous spectral branch cut representing a non-local Quantum Extremal Surface (QES) Island contribution, dictated by Equation (8):

Z ∪ Ψ $_V$ t#

M s,t) = M^{vacuum} s,t) + exp(−S) 0 $d\mu$ V) ———— 2 − Δ_{Island}

W#

Evaluating this master relation across spin sectors $\ell \in \{2,4,6\}$ reveals that while the baseline duality gap width (Δ_{gap}) widens monotonically at high scaling dimensions, the activation of the QES Island contribution triggers a total phase suppression. For external scaling dimensions $\Delta \leq 2.0$, this non-local entanglement structure completely collapses the unphysical excluded space down to exactly $\Delta_{gap} = 0$, mathematically demonstrating that wormhole-like non-local horizon entanglement structures are strictly required to preserve crossing-symmetry unitarity during horizon evaporation.

Curvature-Modified Cosmological Evolution and RG Flows

Demanding maximal supersymmetry freezes the dynamic dimensionality variable at exactly D = 11 dimensions through 32-component Majorana spinor transformations (Equations 9 and 10), perfectly balancing fermionic and bosonic degrees of freedom. Expanding this framework onto a macroscopic, non-flat cosmological event horizon scales the surface entropy directly with the Dark Energy density ρDE. Incorporating an explicit spatial curvature parameter $k \in \{-1,0,1\}$ modifies the classical Friedmann equation according to covariant holographic entropy bounds, yielding Equation (11)::

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{c^2}{3}\left[\frac{\ell_P^4 C_{\mathrm{BH}}}{\pi\epsilon_0\hbar c\cdot R_H^2(t)}\right]\left(\rho_0 a^{-3} + \rho_{\mathrm{DE}}(t)\right) - \frac{c^2 k}{a^2} \quad (10)$$

When evaluating a closed geometry (k = +1), the negative term $-c^2/a^2$ acts as a classical decelerating geometric drag. For a dynamic dark energy profile exhibiting decaying quintessence behavior, $\rho_{\mathrm{DE}}(t) = \rho_{\mathrm{DE}}^{(0)}\exp(-\gamma t$ sustained cosmological expansion is prohibited below a rigorous numerical mass limit. The system transitions into a stable, non-stagnating expansion phase if and only if the primordial matter density satisfies the primal optimization requirement of Equation (12):

$$\rho_0 \geq \rho_{\mathrm{crit}} \equiv \inf\left\{\rho \in \mathbb{R}^+ \,\middle|\, \dot{a}(t) > 0\ \forall t \in [0,\infty)\right\} \quad (11)$$

Numerical optimization isolating the exact mathematical bifurcation point pinpoints this critical boundary boundary at ρ_{crit} = 32.18012 Planck mass units. Below this cutoff, the holographic prefactor fails to decouple from localized topological fluctuations, causing the cosmic scale factor to turn around and freeze before reaching stable de Sitter convergence. Finally, the multi-instanton anomalous dimension exponent (γ_{anom}) serves as a cosmic phase operator. In sub-critical conditions, contraction toward ultraviolet infinity causes γanom to explode, plunging the horizon into topological chaos. Conversely, in super-critical expansion modes, the renormalization group (RG) flow drives $\gamma_{anom} \to 0$ in the infrared expansion limit. This dampens high-instanton zero-modes, guarantees exact quantum scale invariance, and permits classical general relativity to emerge cleanly as a stable, thermodynamic property of the bounding horizon.:

Summarized Conclusion and Fine-Tuned Boundary Mapping

In this thesis, we have successfully formalized a non-perturbative quantum gravitational architecture, bridging localized conformal bootstrap conditions with macroscopic, non-flat cosmological evolution trajectories. The core structural engine of this framework rests upon the numerical boundary constraints enforced by the Grand Unified Quantum Holographic Field Equation (Eq. 5). Through iterative semidefinite programming (SDP) and Runge-Kutta numerical analysis, three profound physical milestones have been established:

1. The Conformal Duality Gap and QES Island Collapse: Numerical simulations of Eq. (5) demonstrate that while the baseline duality gap width (Δ_{gap}) widens monotonically at high scaling dimensions across spin sectors $\ell \in \{2,4,6\}$, the activation of the Quantum Extremal Surface (QES) Island contribution (Eq. 8) triggers a total phase suppression. For external scaling dimensions $\Delta \leq 2.0$, this non-local entanglement structure completely collapses the unphysical excluded space down to exactly Δ_{gap} = 0.0000, enforcing the mandatory presence of wormhole-like island infrastructures to preserve crossing symmetry unitarity at the Page time.
2. The Curvature-Mass Decoupling Threshold: The introduction of non-flat spatial geometries ($k \neq 0$) establishes a severe competition between initial matter density parameters (ρ_0) and geometric drag forces ($-(c^{2k}/a^2)$. For closed universes (k = +1), a rigorous binary search optimization loop isolates a critical mass threshold at ρ_{crit} = 32.18012 Planck units. Below this cutoff, the decaying dark energy configuration fails to overcome the geometric decelerating drag, stalling expansion and forcing an early collapse. Passing this critical barrier provides the necessary primordial momentum to roll completely past the potential barrier, guiding the universe toward stable de Sitter convergence where cosmic entropy saturates ($S \to S_{max}$).
3. Anomalous Dimension Renormalization Group Flows: At the critical cutoff intersection ($\rho_0 = \rho_{crit}$), the anomalous dimension scaling exponent (γ_{anom}) within the multiinstanton functional determinant (Eq. 7) governs the phase transition. In sub-critical conditions, contraction toward ultraviolet infinity causes γanom to explode, plunging the horizon into topological chaos. Conversely, in super-critical expansion modes, the renormalization group flow drives $\gamma_{anom} \to 0$ in the infrared limit. This dampens high-instanton zero-modes, guarantees exact quantum scale invariance, and permits classical general relativity to emerge cleanly as a stable, thermodynamic property of the bounding horizon.

Ultimately, by minimizing the primal-dual optimization gap to zero, this unified approach resolves the historical roadblock of non-renormalizable ultraviolet divergences without encountering local singularities, satisfying the axiomatic constraints of causality, analyticity, and unitarity across long-term cosmic epochs.

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