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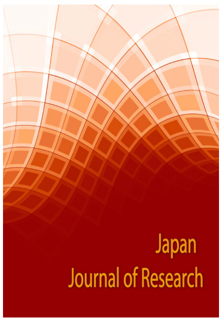

New Electromagnetic Fields and New Gravitational Fields--- Both with Sources Moving at Either Uniform Velocity, or Constant Acceleration, or Continuous Spin

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Abstract

In this article, we show that the vector calculus identities are powerful tool in both rederiving the existing physical laws, such as Maxwell equations, and discovering new physical laws. First, we show that the following physical laws can be rederived from the vector calculus identities: (1) Poynting's Theorem; (2) Gauss's Law for Magnetism; (3) Charge density Continuity equation; (4) Wave equation of potential ***A****, electric field* ***E****, and magnetic field* ***B****; (5) Faraday's Law of Induction; (6) energy density of the magnetic field; (7) time change of energy density of the electric field; (8) time change of energy density of the magsnetic field. To rederive the above physical laws (1) to (8) shows the validation and power of the vector calculus identities in discovering physical laws. Then, applying the vector calculus identities, we derive NEW physical field equations: (1) for electric charges moving with either uniform velocity* ***v****, or constant acceleration* ***a****, or continuous spin* ***S****; (2) for gravitational charges (mass) moving with either uniform velocity* ***v****, or constant acceleration* ***a****, or continuous spin* ***S****. Electromagnetics and gravitoelectromagnetics are derived from the vector calculus identities and have the same form, and thus can be unified. The new physical field equations need to be tested experimentally.*

Introduction

Historically, some of the physical laws were discovered based on experimental data, Experiment → physics law. In this article, we show a different approaching: starting with mathematical identities, then substituting the parameters of physical fields into the mathematical identities to obtain the physical laws, the so-derived laws need to be tested by experiments:

Mathematical equations → physics laws → Experimental testing

In this article, we start with the vector calculus identities which are a branch of mathematics concerned with differentiation of vector fields (gradient, divergence, curl, and combination of above) [1]. We show that vector calculus identities can be used to analyze how physical fields, such as electric fields, magnetic fields, and gravitation fields change in magnitude and direction across space. By substituting the parameters of physical fields into vector calculus identities, we derive the new physical field equations to describe physical fields, such as electric field, magnetic field, and gravity. Some of so-derived physical field equations can be confirmed by existing experiments. New physical field equations need to be confirmed experimentally.

The purpose of this article is to derive new physical laws due to the source moving with different motion status. The magnetic field **B** is created by current density $\mathbf{j} = \rho\mathbf{v}$, where the velocity **v** in vacuum is treated as a constant. By analogy, we list three moving statuses of charges.

Moving statuses of source	Current Density
Stationary	j=0
Uniform velocity: **v**	$j \propto v$
Constant acceleration: a	$j_A \propto \mathbf{A}$
Constant spin: S	$j_S \propto \mathbf{S}$

Note: For the purpose of deriving field equation, we treat the Spin as one of motion status.

Since the velocity of electric charge creates a magnetic field, we propose the following:

1. The acceleration a of electric charge creates magnetic-type fields;
2. The Spin S of electric charge creates magnetic-type fields;
3. The velocity v of gravitational charge creates gravitomagnetic fields;
4. The acceleration a of gravitational charges creates gravitomagnetic-type fields;
5. The Spin S of gravitational charge creates

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gravitomagnetic-type fields.

In Section 2, we list the vector calculus identities used in the article.

In Section 3, we apply vector calculus identities to electromagnetics, (1) rederive the Poynting's theorem, and derive new physical laws; (2) derive new extended electric field equations and magnetic field equations due to the velocity, due to the acceleration, and due to the Spin, respectively.

In Section 4, following the same approach of Section 3, we utilize the concept of the gravitoelectromagnetic field, then apply vector calculus identities to gravitoelectromagnetic fields, and derive new field equation for them.

Vector Calculus Identities

In Section 2, we list 11 vector calculus identities. Then in Section 3 to Section 6, we utilized them to derive existing and new field equations.

(1). Divergence of cross product of two vector fields: Y×Z

$$\nabla\cdot(\mathbf{Y}\times\mathbf{Z})=(\nabla\times\mathbf{Y})\cdot\mathbf{Z}-\mathbf{Y}\cdot(\nabla\times\mathbf{Z}) \quad (1)$$

(2) Curl of cross product of two vector fields: Y×Z

$$\nabla\times(\mathbf{Y}\times\mathbf{Z})=\mathbf{Y}(\nabla\cdot\mathbf{Z})-\mathbf{Z}(\nabla\cdot\mathbf{Y})+(\mathbf{Z}\cdot\nabla)\mathbf{Y}-(\mathbf{Y}\cdot\nabla)\mathbf{Z} \quad (2)$$

(3) Divergence of Curl of vector field: ∇×**Y**

$$\nabla\cdot(\nabla\times\mathbf{Y})=0 \quad (3)$$

(4). Curl of curl of vector field: Y

$$\nabla\times(\nabla\times\mathbf{Y})=\nabla(\nabla\cdot\mathbf{Y})-\nabla^2\,\mathbf{Y} \quad (4)$$

(5) Divergence of Gradient of scalar field: φ

$$\nabla\cdot(\nabla\varphi)=\nabla^2\,\varphi \quad (5)$$

(6) Curl of Gradient of scalar field: φ

$$\nabla\times(\nabla\varphi)=0 \quad (6)$$

(7). Divergence of product of scalar field and vector field: φY

$$\nabla\cdot(\varphi\mathbf{Y})=\varphi(\nabla\cdot\mathbf{Y})+\mathbf{Y}\cdot(\nabla\varphi) \quad (7)$$

(8). Curl of product of scalar field and vector field: φY

$$\nabla\times(\varphi\mathbf{Y})=\varphi(\nabla\times\mathbf{Y})+(\nabla\varphi)\times\mathbf{Y} \quad (8)$$

(9) Gradient of dot product of two vector fields: (Y·Z).

$$\nabla(\mathbf{Y}\cdot\mathbf{Z})=(\mathbf{Y}\cdot\nabla)\mathbf{Z}+(\mathbf{Z}\cdot\nabla)\mathbf{Y}+\mathbf{Y}\times(\nabla\times\mathbf{Z})+\mathbf{Z}\times(\nabla\times\mathbf{Y}) \quad (9)$$

(10) Cross Product: (**Y**×∇)·**Z**

$$(\mathbf{Y}\times\nabla)\cdot\mathbf{Z}=\mathbf{Y}\cdot(\nabla\times\mathbf{Z}) \quad (10)$$

(11) Cross Product: (**Y**×∇)×**Z**

$$(\mathbf{Y}\times\nabla)\times\mathbf{Z}=\mathbf{Y}\times(\nabla\times\mathbf{Z})+(\mathbf{Y}\cdot\nabla)\mathbf{Z}-\mathbf{Y}(\nabla\cdot\mathbf{Z}) \quad (11)$$

Mathematically Deriving Classical Electromagnetics and Novel Electromagnetics

Classical Electromagnetics

AIn Section 3, let Y and Z represent either electric field E, or magnetic field B, or vector potential A, respectively, and φ is scalar potential in Equations (5) to Equation (8). Then substituting E, B, A, and φ into vector calculus identities, we obtain the field equations, respectively.

Divergence of cross product of vector fields: ∇·(E×**B**), ∇·(**A**×**B**), **And** ∇·(**A**×E)

$$\nabla\cdot(\mathbf{Y}\times\mathbf{Z})=(\nabla\times\mathbf{Y})\cdot\mathbf{Z}-\mathbf{Y}\cdot(\nabla\times\mathbf{Z}) \quad (1)$$

Poynting's Theorem: ∇·(E×**B**)

Substituting Y = E and Z = B, Equation (1) gives Poynting's Theorem:

$$\nabla\cdot(E\times\mathbf{B})=(\nabla\times E)\cdot\mathbf{B}-E\cdot(\nabla\times\mathbf{B})=-1/2\,\partial(E^2+\mathbf{B}^2)/\partial\mathbf{t}-E\cdot j \quad (3.1)$$

Where $\nabla\times\mathbf{B}=j+(\partial E/\partial\mathbf{t})$ and $\nabla\times E=-(\partial\mathbf{B}/\partial\mathbf{t})$ are used. The term E×B is the Poynting vector representing the energy flux (power per unit area) of the electromagnetic field. The term, $\partial(E^2+B^2)/\partial t$, is the rate of change of the stored electromagnetic energy density. The term E·J represents the rate at which the electromagnetic field does work on the charge carriers, and **J=ρv.**

New field equation: ∇·(**A**×**B**) **And** ∇·(**A**×E)

For completeness, let consider vector potential A as well. We obtain new field equations.

Substituting (Y = A, Z = B) and (Y = A, Z = E), respectively, Equation (1) gives NEW field equation:

$$\nabla\cdot(\mathbf{A}\times\mathbf{B})=(\nabla\times\mathbf{A})\cdot\mathbf{B}-\mathbf{A}\cdot(\nabla\times\mathbf{B})=\mathbf{B}\cdot\mathbf{B}-\mathbf{A}\cdot j-\mathbf{A}\cdot(\partial E/\partial\mathbf{t}). \quad (3.2)$$

$$\nabla\cdot(\mathbf{A}\times E)=(\nabla\times\mathbf{A})\cdot E-\mathbf{A}\cdot(\nabla\times E)=E\cdot\mathbf{B}+\mathbf{A}\cdot(\partial\mathbf{B}/\partial\mathbf{t}). \quad (3.3)$$

Equation (3.2) represent the energy of magnetic field.

Curl of cross product of two vector fields: ∇×(E×**B**), ∇×(**A**×**B**), and ∇×(**A**×E)

$$\nabla\times(\mathbf{Y}\times\mathbf{Z})=\mathbf{Y}(\nabla\cdot\mathbf{Z})-\mathbf{Z}(\nabla\cdot\mathbf{Y})+(\mathbf{Z}\cdot\nabla)\mathbf{Y}-(\mathbf{Y}\cdot\nabla)\mathbf{Z} \quad (2)$$

Substituting (Y = E, Z = B), (Y = A, Z = B), and (Y = A, Z = E), respectively, Equation (2) gives

NEW field equations:

$$\nabla\times(E\times\mathbf{B})=E(\nabla\cdot\mathbf{B})-\mathbf{B}(\nabla\cdot E)+(\mathbf{B}\cdot\nabla)E-(E\cdot\nabla)\mathbf{B}$$
$$=-\rho\mathbf{B}+(\mathbf{B}\cdot\nabla)E-(E\cdot\nabla)\mathbf{B} \quad (3.4)$$

$$\nabla\times(\mathbf{A}\times\mathbf{B})=\mathbf{A}(\nabla\cdot\mathbf{B})-\mathbf{B}(\nabla\cdot\mathbf{A})+(\mathbf{B}\cdot\nabla)\mathbf{A}-(\mathbf{A}\cdot\nabla)\mathbf{B}$$
$$=-\mathbf{B}(\nabla\cdot\mathbf{A})+(\mathbf{B}\cdot\nabla)\mathbf{A}-(\mathbf{A}\cdot\nabla)\mathbf{B} \quad (3.5)$$

$$\nabla\times(\mathbf{A}\times E)=\mathbf{A}(\nabla\cdot E)-E(\nabla\cdot\mathbf{A})+(E\cdot\nabla)\mathbf{A}-(\mathbf{A}\cdot\nabla)E$$
$$=\rho\mathbf{A}-E(\nabla\cdot\mathbf{A})+(E\cdot\nabla)\mathbf{A}-(\mathbf{A}\cdot\nabla)E\quad. \quad (3.6)$$

Where ∇·E=ρ.

Divergence of Curl of vector field: ∇·(∇×**A**), ∇·(∇×E), and ∇·(∇×**B**)

$$\nabla\cdot(\nabla\times\mathbf{Y})=0 \quad (3)$$

Gauss's Law for Magnetism (1): ∇·(∇×**A**)

Substituting Y = A and the definition of the magnetic field, **B**=∇×**A**, Equation (3) gives Gauss's Law for Magnetism:

$$\nabla\cdot(\nabla\times\mathbf{A})=\nabla\cdot\mathbf{B}=0 \quad (3.7)$$

Equation (3.7) mathematically indicates no magnetic monopoles.

Gauss's Law for Magnetism (2): ∇·(∇×E)

Substituting Y = E, Equation (3) gives:

$$\nabla\cdot(\nabla\times E)=0 \quad (3.8)$$

Substituting the Faraday's Law of Induction into Equation (3.8), we obtain:

$$\nabla\cdot(\nabla\times E)=-\nabla\cdot(\partial\mathbf{B}/\partial\mathbf{t})=-\partial/\partial\mathbf{t}\,(\nabla\cdot\mathbf{B})=0. \quad (3.9a)$$

Equation (3.9a) has two solutions:

(1). The magnetic field B is a constant field:

$$\partial B/\partial t=0. \quad (3.9b)$$

(2). Gauss's Law for Magnetism

$$\nabla\cdot\mathbf{B}(\mathbf{t})=0. \quad (3.9c)$$

Equation (3.9a) is valid for both situations:

(1) constant magnetic field, Equation (3.9b); and

(2) time-varying magnetic field Equation (3.9c).

Charge density Continuity equation: $\nabla\cdot(\nabla\times\mathbf{B})$

Substituting Y = B, Equation (3) gives:

$$\nabla\cdot(\nabla\times\mathbf{B})=0 \tag{3.10}$$

Substituting the Ampere-Maxwell equation into Equation (3.10), we obtain Charge density Continuity equation,

$$\nabla\cdot(\nabla\times\mathbf{B})=\nabla\cdot j+\nabla\cdot\partial E/\partial\mathbf{t}=\nabla\cdot j+\partial/\partial\mathbf{t}\,\rho=0 \tag{3.11}$$

Curl of curl of vector field: wave equation: $\nabla\times(\nabla\times\mathbf{A})$, $\nabla\times(\nabla\times E)$, **And** $\nabla\times(\nabla\times\mathbf{B})$

$$\nabla\times(\nabla\times\mathbf{Y})=\nabla(\nabla\cdot\mathbf{Y})-\nabla^2\,\mathbf{Y} \tag{4}$$

Wave equation of vector potential A: $\nabla\times(\nabla\times\mathbf{A})$

Substituting Y = A, Equation (4) gives:

$$\nabla\times(\nabla\times\mathbf{A})=\nabla\times\mathbf{B}=j+\partial/\partial\mathbf{t}\,E \tag{3.12a}$$

and

$$\nabla(\nabla\cdot\mathbf{A})-\nabla^2\,\mathbf{A}=-\partial/\partial\mathbf{t}\,(\nabla\varphi)-\nabla^2\,\mathbf{A}=\partial/\partial\mathbf{t}\,E+\partial^2/(\partial\mathbf{t}^2)\,\mathbf{A}-\nabla^2\,\mathbf{A} \tag{3.12b}$$

Combining Equation (3.12a) and Equation (3.12b), we obtain wave equation of vector potential A:

$$\partial^2/(\partial\mathbf{t}^2)\,\mathbf{A}-\nabla^2\,\mathbf{A}=j \tag{3.12c}$$

Where the definition of E field is applied: $E=-\nabla\varphi-\partial/\partial\mathbf{t}\,\mathbf{A}$.

Wave equation of electric field E: $\nabla\times(\nabla\times E)$

Substituting Y = E, Equation (4) gives

$$\nabla\times(\nabla\times E)=-\nabla\times\partial/\partial\mathbf{t}\,\mathbf{B}=-\partial/\partial\mathbf{t}\,j-\partial^2/(\partial\mathbf{t}^2)\,E \tag{3.13a}$$

And

$$\nabla(\nabla\cdot E)-\nabla^2\,E=\nabla\rho-\nabla^2\,E \tag{3.13b}$$

Combining Equation (3.13a) and Equation (3.13b), we obtain wave equation of electric field E,

$$\partial^2/(\partial\mathbf{t}^2)\,E-\nabla^2\,E=-\partial/\partial\mathbf{t}\,j-\nabla\rho \tag{3.13c}$$

Wave equation of magnetic field B: $\nabla\times(\nabla\times\mathbf{B})$

Substituting Y = B, Equation (4) gives

$$\nabla\times(\nabla\times\mathbf{B})=\nabla\times j+\nabla\times\partial/\partial\mathbf{t}\,E=\nabla\times j-\partial^2/(\partial\mathbf{t}^2)\,\mathbf{B} \tag{3.14a}$$

And

$$\nabla(\nabla\cdot\mathbf{B})-\nabla^2\,\mathbf{B}=-\nabla^2\,\mathbf{B} \tag{3.14b}$$

Combining Equation (3.14a) and Equation (3.14b), we obtain wave equation of B,

$$\partial^2/(\partial\mathbf{t}^2)\,\mathbf{B}-\nabla^2\,\mathbf{B}=\nabla\times j \tag{3.14c}$$

Divergence of Gradient of scalar field: wave equation of scalar potential φ: $\nabla\cdot(\nabla\varphi)$

$$\nabla\cdot(\nabla\varphi)=\nabla^2\,\varphi \tag{5}$$

Substituting the definition of electric field, $E=-\nabla\varphi-\partial/\partial\mathbf{t}\,\mathbf{A}$ into Equation (5). We obtain wave equation of potential φ:

$$\nabla\cdot(\nabla\varphi)=\nabla\cdot(-E-\partial/\partial\mathbf{t}\,\mathbf{A})=-\rho+\partial^2/(\partial\mathbf{t}^2)\,\varphi \tag{3.15a}$$

i.e.,

$$\partial^2/(\partial\mathbf{t}^2)\,\varphi-\nabla^2\,\varphi=\rho. \tag{3.15b}$$

Curl of Gradient of scalar field: Faraday's Law of Induction: $\nabla\times(\nabla\varphi)$

$$\nabla\times(\nabla\varphi)=0 \tag{6}$$

Substituting the definition of the electric field, $E=-\nabla\varphi-\partial\mathbf{A}/\partial\mathbf{t}$, into Equation (6).

We obtain Faraday's Law of Induction:

$$\nabla\times(\nabla\varphi)=\nabla\times(-E-\partial\mathbf{A}/\partial\mathbf{t})=-\nabla\times E-\partial\mathbf{B}/\partial\mathbf{t}=0. \tag{3.16a}$$

i.e.,

$$\nabla\times E=-\partial\mathbf{B}/\partial\mathbf{t}. \tag{3.16b}$$

Divergence of product of scalar potential φ with vector potential A, electric field E, magnetic field B:

$$\nabla\cdot(\varphi\mathbf{Y})=\varphi(\nabla\cdot\mathbf{Y})+\mathbf{Y}\cdot(\nabla\varphi) \tag{7}$$

Substituting Y = A, Y = E, and Y = B into Equation (7), respectively, we obtain NEW field equations:

$$\nabla\cdot(\varphi\mathbf{A})=\varphi(\nabla\cdot\mathbf{A})+\mathbf{A}\cdot(\nabla\varphi) \tag{3.17}$$

$$\nabla\cdot(\varphi E)=\varphi(\nabla\cdot E)+E\cdot(\nabla\varphi)=\rho\varphi+E\cdot(\nabla\varphi) \tag{3.18}$$

$$\nabla\cdot(\varphi\mathbf{B})=\varphi(\nabla\cdot\mathbf{B})+\mathbf{B}\cdot(\nabla\varphi)=\mathbf{B}\cdot(\nabla\varphi) \tag{3.19}$$

Curl of product of scalar potential φ and vector field: $\nabla\times(\varphi\mathbf{A})$, $\nabla\times(\varphi E)$, **And** $\nabla\times(\varphi\mathbf{B})$

$$\nabla\times(\varphi\mathbf{Y})=\varphi(\nabla\times\mathbf{Y})+(\nabla\varphi)\times\mathbf{Y}. \tag{8}$$

Substituting Y = A, Y = E, and Y = B into Equation (8), respectively, we obtain NEW field equations:

$$\nabla\times(\varphi\mathbf{A})=\varphi(\nabla\times\mathbf{A})+(\nabla\varphi)\times\mathbf{A}=\varphi\mathbf{B}+(\nabla\varphi)\times\mathbf{A} \tag{3.20}$$

$$\nabla\times(\varphi E)=\varphi(\nabla\times E)+(\nabla\varphi)\times E=-\varphi\partial\mathbf{B}/\partial\mathbf{t}+(\nabla\varphi)\times E \tag{3.21}$$

$$\nabla\times(\varphi\mathbf{B})=\varphi(\nabla\times\mathbf{B})+(\nabla\varphi)\times\mathbf{B}=j\varphi+\varphi\partial E/\partial\mathbf{t}+(\nabla\varphi)\times\mathbf{B} \tag{3.22}$$

Gradient of scalar product of two vector fields: $\nabla(E\cdot\mathbf{B})$, $\nabla(\mathbf{A}\cdot\mathbf{B})$, $\nabla(\mathbf{A}\cdot E)$, $\nabla(\mathbf{A}\cdot\mathbf{A})$, $\nabla(E\cdot E)$, $\nabla(\mathbf{B}\cdot\mathbf{B})$

$$\nabla(\mathbf{Y}\cdot\mathbf{Z})=(\mathbf{Y}\cdot\nabla)\mathbf{Z}+(\mathbf{Z}\cdot\nabla)\mathbf{Y}+\mathbf{Y}\times(\nabla\times\mathbf{Z})+\mathbf{Z}\times(\nabla\times\mathbf{Y}). \tag{9}$$

Substituting (Y = E, Z = B), (Y = A, Z=B), (Y=A, Z=E), (Y = Z = B), (Y = Z =A), Y=Z=E), into Equation (9), respectively, we obtain NEW field equations:

$$\nabla(E\cdot\mathbf{B})=(E\cdot\nabla)\mathbf{B}+(\mathbf{B}\cdot\nabla)E+E\times(\nabla\times\mathbf{B})+\mathbf{B}\times(\nabla\times E)$$
$$=(E\cdot\nabla)\mathbf{B}+(\mathbf{B}\cdot\nabla)E+E\times j \tag{3.23}$$

$$\nabla(\mathbf{A}\cdot\mathbf{B})=(\mathbf{A}\cdot\nabla)\mathbf{B}+(\mathbf{B}\cdot\nabla)\mathbf{A}+\mathbf{A}\times(\nabla\times\mathbf{B})+\mathbf{B}\times(\nabla\times\mathbf{A})$$
$$=(\mathbf{A}\cdot\nabla)\mathbf{B}+(\mathbf{B}\cdot\nabla)\mathbf{A}+\mathbf{A}\times j+\mathbf{A}\times\partial E/\partial\mathbf{t} \tag{3.24}$$

$$\nabla(\mathbf{A}\cdot E)=(\mathbf{A}\cdot\nabla)E+(E\cdot\nabla)\mathbf{A}+\mathbf{A}\times(\nabla\times E)+E\times(\nabla\times\mathbf{A})$$
$$=(\mathbf{A}\cdot\nabla)E+(E\cdot\nabla)\mathbf{A}-\mathbf{A}\times\partial\mathbf{B}/\partial\mathbf{t}+E\times\mathbf{B} \tag{3.25}$$

$$\nabla(\mathbf{A}\cdot\mathbf{A})=2(\mathbf{A}\cdot\nabla)\mathbf{A}+2\mathbf{A}\times(\nabla\times\mathbf{A})$$
$$=2(\mathbf{A}\cdot\nabla)\mathbf{A}+2\mathbf{A}\times\mathbf{B} \tag{3.26}$$

$$\nabla(E\cdot E)=(E\cdot\nabla)E+(E\cdot\nabla)E+E\times(\nabla\times E)+E\times(\nabla\times E)$$
$$=2(E\cdot\nabla)E-2E\times(\partial\mathbf{B}/\partial\mathbf{t}) \tag{3.27}$$

$$\nabla(\mathbf{B}\cdot\mathbf{B})=(\mathbf{B}\cdot\nabla)\mathbf{B}+(\mathbf{B}\cdot\nabla)\mathbf{B}+\mathbf{B}\times(\nabla\times\mathbf{B})+\mathbf{B}\times(\nabla\times\mathbf{B})$$
$$=2(\mathbf{B}\cdot\nabla)\mathbf{B}+2\mathbf{B}\times(\nabla\times\mathbf{B}). \tag{3.28}$$

Cross Product: $(E\times\nabla)\cdot\mathbf{B}$, $(\mathbf{B}\times\nabla)\cdot E$, $(\mathbf{A}\times\nabla)\cdot\mathbf{B}$, $(\mathbf{B}\times\nabla)\cdot\mathbf{A}$, $(E\times\nabla)\cdot\mathbf{A}$, **And** $(\mathbf{A}\times\nabla)\cdot E$

Equ**At**ion (10),

$$(\mathbf{Y}\times\nabla)\cdot\mathbf{Z}=\mathbf{Y}\cdot(\nabla\times\mathbf{Z}) \tag{10}$$

Substituting (Y=E and Z=B), (Y=B and Z=E), (Y=A and Z=B), (Y=B and Z=A), (Y=E and Z=A), and (Y=A and Z=E), respectively, Equation (10) gives the following equations,

$$(E\times\nabla)\cdot\mathbf{B}=E\cdot(\nabla\times\mathbf{B})=E\cdot j+E\cdot\partial E/\partial\mathbf{t} \tag{3.29}$$

$$(\mathbf{B}\times\nabla)\cdot E=\mathbf{B}\cdot(\nabla\times E)=-\mathbf{B}\cdot\partial\mathbf{B}/\partial\mathbf{t}. \tag{3.30}$$

$$(\mathbf{A}\times\nabla)\cdot\mathbf{B}=\mathbf{A}\cdot(\nabla\times\mathbf{B})=\mathbf{A}\cdot j+\mathbf{A}\cdot\partial E/\partial\mathbf{t} \tag{3.31}$$

$(\mathbf{B}\times\nabla)\cdot\mathbf{A}=\mathbf{B}\cdot(\nabla\times\mathbf{A})=\mathbf{B}^2$ (3.32)

$(\mathrm{E}\times\nabla)\cdot\mathbf{A}=\mathrm{E}\cdot(\nabla\times\mathbf{A})=\mathrm{E}\cdot\mathbf{B}$ (3.33)

$(\mathbf{A}\times\nabla)\cdot\mathrm{E}=\mathbf{A}\cdot(\nabla\times\mathrm{E})=-\mathbf{A}\cdot\partial\mathbf{B}/\partial\mathbf{t}$ (3.34)

Cross Product: $(\mathrm{E}\times\nabla)\times\mathbf{B}$, $(\mathbf{B}\times\nabla)\times\mathrm{E}$, $(\mathbf{A}\times\nabla)\times\mathbf{B}$, $(\mathbf{B}\times\nabla)\times\mathbf{A}$, $(\mathbf{A}\times\nabla)\times\mathrm{E}$, **And** $(\mathrm{E}\times\nabla)\times\mathbf{A}$

$(\mathbf{Y}\times\nabla)\times\mathbf{Z}=\mathbf{Y}\times(\nabla\times\mathbf{Z})+(\mathbf{Y}\cdot\nabla)\mathbf{Z}-\mathbf{Y}(\nabla\cdot\mathbf{Z})$ (11)

Substituting (Y=E and Z=B), (Y=B and Z=E), (Y=A and Z=B), (Y=B and Z=A), (Y=A and Z=E), and (Y=E and Z=A), Equation (11) gives new field equations:

$(\mathrm{E}\times\nabla)\times\mathbf{B}=\mathrm{E}\times(\nabla\times\mathbf{B})+(\mathrm{E}\cdot\nabla)\mathbf{B}-\mathrm{E}(\nabla\cdot\mathbf{B})\ =\mathrm{E}\times\mathrm{j}+\mathrm{E}\times\partial\mathrm{E}/\partial\mathbf{t}+(\mathrm{E}\cdot\nabla)\mathbf{B}$

$=\mathrm{E}\times\mathrm{j}+(\mathrm{E}\cdot\nabla)\mathbf{B}$ (3.35)

$(\mathbf{B}\times\nabla)\times\mathrm{E}=\mathbf{B}\times(\nabla\times\mathrm{E})+(\mathbf{B}\cdot\nabla)\mathrm{E}-\mathbf{B}(\nabla\cdot\mathrm{E})=-\mathbf{B}\times\partial\mathbf{B}/\partial\mathbf{t}+(\mathbf{B}\cdot\nabla)\mathrm{E}-\rho\mathbf{B}=-\rho\mathbf{B}+(\mathbf{B}\cdot\nabla)\mathrm{E}$ (3.36)

$(\mathbf{A}\times\nabla)\times\mathbf{B}=\mathbf{A}\times(\nabla\times\mathbf{B})+(\mathbf{A}\cdot\nabla)\mathbf{B}-\mathbf{A}(\nabla\cdot\mathbf{B})$

$=\mathbf{A}\times\mathrm{j}+\mathbf{A}\times\partial\mathrm{E}/\partial\mathbf{t}+(\mathbf{A}\cdot\nabla)\mathbf{B}$ (3.37)

$(\mathbf{B}\times\nabla)\times\mathbf{A}=\mathbf{B}\times(\nabla\times\mathbf{A})+(\mathbf{B}\cdot\nabla)\mathbf{A}-\mathbf{B}(\nabla\cdot\mathbf{A})$

$=(\mathbf{B}\cdot\nabla)\mathbf{A}+\mathbf{B}(\partial\varphi/\partial\mathbf{t})$ (3.38)

$(\mathbf{A}\times\nabla)\times\mathrm{E}=\mathbf{A}\times(\nabla\times\mathrm{E})+(\mathbf{A}\cdot\nabla)\mathrm{E}-\mathbf{A}(\nabla\cdot\mathrm{E})$

$=-\mathbf{A}\times\partial\mathbf{B}/\partial\mathbf{t}+(\mathbf{A}\cdot\nabla)\mathrm{E}-\rho\mathbf{A}$ (3.39)

$(\mathrm{E}\times\nabla)\times\mathbf{A}=\mathrm{E}\times\mathbf{B}+(\mathrm{E}\cdot\nabla)\mathbf{A}-\mathrm{E}(\nabla\cdot\mathbf{A})$

$=\mathrm{E}\times\mathbf{B}+(\mathbf{B}\cdot\nabla)\mathbf{A}+\mathrm{E}(\partial\varphi/\partial\mathbf{t})$ (3.40)

Electric field and magnetic field created by electric charge moving with velocity v

(3.2.1). Divergence of cross product of velocity v and vector fields: $\nabla\cdot(\mathrm{v}\times\mathbf{A})$, $\nabla\cdot(\mathrm{v}\times\mathrm{E})$, **And** $\nabla\cdot(\mathrm{v}\times\mathbf{B})$

$\nabla\cdot(\mathbf{Y}\times\mathbf{Z})=(\nabla\times\mathbf{Y})\cdot\mathbf{Z}-\mathbf{Y}\cdot(\nabla\times\mathbf{Z})$ (1)

Substituting (Y = v, Z = A), (Y = v, Z = E), and (Y = v, Z = B), respectively, Equation (1) gives New field equations:

$\nabla\cdot(\mathrm{v}\times\mathbf{A})=(\nabla\times\mathrm{v})\cdot\mathbf{A}-\mathrm{v}\cdot(\nabla\times\mathbf{A})=-\mathrm{v}\cdot\mathbf{B}$. (3.41)

$\nabla\cdot(\mathrm{v}\times\mathrm{E})=(\nabla\times\mathrm{v})\cdot\mathrm{E}-\mathrm{v}\cdot(\nabla\times\mathrm{E})=\mathrm{v}\cdot\partial(\mathbf{B})/\partial\mathbf{t}$. (3.42)

$\nabla\cdot(\mathrm{v}\times\mathbf{B})=(\nabla\times\mathrm{v})\cdot\mathbf{B}-\mathrm{v}\cdot(\nabla\times\mathbf{B})=-\mathrm{v}\cdot\mathrm{j}-\mathrm{v}\cdot\partial(\mathrm{E})/\partial\mathbf{t}$ (3.43)

Where v is particles' average drift velocity.

Curl of cross product of velocity v and vector fields: $\nabla\times(\mathrm{v}\times\mathbf{A})$, $\nabla\times(\mathrm{v}\times\mathrm{E})$, $\nabla\times(\mathrm{v}\times\mathbf{B})$

$\nabla\times(\mathbf{Y}\times\mathbf{Z})=\mathbf{Y}(\nabla\cdot\mathbf{Z})-\mathbf{Z}(\nabla\cdot\mathbf{Y})+(\mathbf{Z}\cdot\nabla)\mathbf{Y}-(\mathbf{Y}\cdot\nabla)\mathbf{Z}$ (2)

Substituting (Y = v, Z = A), (Y = v, Z = E), and (Y = v, Z = B), respectively, Equation (2) gives NEW field equations:

$\nabla\times(\mathrm{v}\times\mathbf{A})=\mathrm{v}(\nabla\cdot\mathbf{A})-\mathbf{A}(\nabla\cdot\mathrm{v})+(\mathbf{A}\cdot\nabla)\mathrm{v}-(\mathrm{v}\cdot\nabla)\mathbf{A}=\mathrm{v}(\nabla\cdot\mathbf{A})-\partial(\mathbf{A})/\partial\mathbf{t}$. (3.44)

$\nabla\times(\mathrm{v}\times\mathrm{E})=\mathrm{v}(\nabla\cdot\mathrm{E})-\mathrm{E}(\nabla\cdot\mathrm{v})+(\mathrm{E}\cdot\nabla)\mathrm{v}-(\mathrm{v}\cdot\nabla)\mathrm{E}=\rho\mathrm{v}-\partial(\mathrm{E})/\partial\mathbf{t}$ (3.45)

$\nabla\times(\mathrm{v}\times\mathbf{B})=\mathrm{v}(\nabla\cdot\mathbf{B})-\mathbf{B}(\nabla\cdot\mathrm{v})+(\mathbf{B}\cdot\nabla)\mathrm{v}-(\mathrm{v}\cdot\nabla)\mathbf{B}=-\partial(\mathbf{B})/\partial\mathbf{t}$. (3.46)

Divergence of product of scalar potential φ and velocity v: $\nabla\cdot(\varphi\mathrm{v})$

$\nabla\cdot(\varphi\mathbf{Y})=\varphi(\nabla\cdot\mathbf{Y})+\mathbf{Y}\cdot(\nabla\varphi)$ (7)

Substituting Y = v into Equation (7), we obtain NEW field equation

$\nabla\cdot(\varphi\mathrm{v})=\varphi(\nabla\cdot\mathrm{v})+\mathrm{v}\cdot(\nabla\varphi)=\mathrm{v}\cdot(\nabla\varphi)$ (3.47)

Curl of product of scalar field φ and velocity v: $\nabla\times(\varphi\mathrm{v})$

$\nabla\times(\varphi\mathbf{Y})=\varphi(\nabla\times\mathbf{Y})+(\nabla\varphi)\times\mathbf{Y}$. (8)

Substituting Y = v into Equation (8), we obtain NEW field equation:

$\nabla\times(\varphi\mathrm{v})=\varphi(\nabla\times\mathrm{v})+(\nabla\varphi)\times\mathrm{v}=(\nabla\varphi)\times\mathrm{v}$ (3.48)

Gradient of scalar product of velocity v and vector fields: $\nabla(\mathrm{v}\cdot\mathbf{A})$, $\nabla(\mathrm{v}\cdot\mathrm{E})$, **And** $\nabla(\mathrm{v}\cdot\mathbf{B})$

$\nabla(\mathbf{Y}\cdot\mathbf{Z})=(\mathbf{Y}\cdot\nabla)\mathbf{Z}+(\mathbf{Z}\cdot\nabla)\mathbf{Y}+\mathbf{Y}\times(\nabla\times\mathbf{Z})+\mathbf{Z}\times(\nabla\times\mathbf{Y})$. (9)

Substituting (Y = v, Z=A), or (Y = v, Z=E), or (Y = v, Z=B), into Equation (9), respectively, we obtain NEW field equations:

$\nabla(\mathrm{v}\cdot\mathbf{A})=(\mathrm{v}\cdot\nabla)\mathbf{A}+(\mathbf{A}\cdot\nabla)\mathrm{v}+\mathrm{v}\times(\nabla\times\mathbf{A})+\mathbf{A}\times(\nabla\times\mathrm{v})$

$=(\mathrm{v}\cdot\nabla)\mathbf{A}+\mathrm{v}\times\mathbf{B}$ (3.49)

$\nabla(\mathrm{v}\cdot\mathrm{E})=(\mathrm{v}\cdot\nabla)\mathrm{E}+(\mathrm{E}\cdot\nabla)\mathrm{v}+\mathrm{v}\times(\nabla\times\mathrm{E})+\mathrm{E}\times(\nabla\times\mathrm{v})$

$=(\mathrm{v}\cdot\nabla)\mathrm{E}+\mathrm{v}\times(\nabla\times\mathrm{E})$. (3.50)

$\nabla(\mathrm{v}\cdot\mathbf{B})=(\mathrm{v}\cdot\nabla)\mathbf{B}+(\mathbf{B}\cdot\nabla)\mathrm{v}+\mathrm{v}\times(\nabla\times\mathbf{B})+\mathbf{B}\times(\nabla\times\mathrm{v})$

$=(\mathrm{v}\cdot\nabla)\mathbf{B}+\mathrm{v}\times(\nabla\times\mathbf{B})$. (3.51)

Cross Product: $(\mathrm{v}\times\nabla)\cdot\mathbf{A}$, $(\mathrm{v}\times\nabla)\cdot\mathrm{E}$, **And** $(\mathrm{v}\times\nabla)\cdot\mathbf{B}$

$(\mathbf{Y}\times\nabla)\cdot\mathbf{Z}=\mathbf{Y}\cdot(\nabla\times\mathbf{Z})$. (10)

Substituting (Y=v, Z=A), or (Y=v,Z=E), or (Y=v,Z=B), respectively, Equation (10) gives new field equations,

$(\mathrm{v}\times\nabla)\cdot\mathbf{A}=\mathrm{v}\cdot(\nabla\times\mathbf{A})=\mathrm{v}\cdot\mathbf{B}$. (3.52)

$(\mathrm{v}\times\nabla)\cdot\mathrm{E}=\mathrm{v}\cdot(\nabla\times\mathrm{E})=-\mathrm{v}\cdot\partial\mathbf{B}/\partial\mathbf{t}$. (3.53)

$(\mathrm{v}\times\nabla)\cdot\mathbf{B}=\mathrm{v}\cdot(\nabla\times\mathbf{B})=\mathrm{v}\cdot\mathrm{j}+\mathrm{v}\cdot\partial\mathrm{E}/\partial\mathbf{t}$. (3.54)

Cross Product: $(\mathrm{v}\times\nabla)\times\mathbf{A}$, $(\mathrm{v}\times\nabla)\times\mathrm{E}$, **And** $(\mathrm{v}\times\nabla)\times\mathbf{B}$

$(\mathbf{Y}\times\nabla)\times\mathbf{Z}=\mathbf{Y}\times(\nabla\times\mathbf{Z})+(\mathbf{Y}\cdot\nabla)\mathbf{Z}-\mathbf{Y}(\nabla\cdot\mathbf{Z})$ (11)

Substituting (Y=v, Z=A), or (Y=v,Z=E), or (Y=v,Z=B), respectively, Equation (11) gives new field equations:

$(\mathrm{v}\times\nabla)\times\mathbf{A}=\mathrm{v}\times(\nabla\times\mathbf{A})+(\mathrm{v}\cdot\nabla)\mathbf{A}-\mathrm{v}(\nabla\cdot\mathbf{A})$

$=\mathrm{v}\times\mathbf{B}+\partial\mathbf{A}/\partial\mathbf{t}-\mathrm{v}(\nabla\cdot\mathbf{A})$. (3.55)

$(\mathrm{v}\times\nabla)\times\mathrm{E}=\mathrm{v}\times(\nabla\times\mathrm{E})+(\mathrm{v}\cdot\nabla)\mathrm{E}-\mathrm{v}(\nabla\cdot\mathrm{E})$

$=\mathrm{v}\times(\nabla\times\mathrm{E})+\partial\mathrm{E}/\partial\mathbf{t}-\rho\mathrm{v}$. (3.56)

$(\mathrm{v}\times\nabla)\times\mathbf{B}=\mathrm{v}\times(\nabla\times\mathbf{B})+(\mathrm{v}\cdot\nabla)\mathbf{B}-\mathrm{v}(\nabla\cdot\mathbf{B})$

$=\mathrm{v}\times(\nabla\times\mathbf{B})+\partial\mathbf{B}/\partial\mathbf{t}$. (3.57)

Electric field and magnetic field created by electric charge moving with acceleration a

Divergence of cross product of acceleration a and vector fields: $\nabla\cdot(\mathbf{A}\times\mathbf{A})$, $\nabla\cdot(\mathbf{A}\times\mathrm{E})$, **And** $\nabla\cdot(\mathbf{A}\times\mathbf{B})$

$\nabla\cdot(\mathbf{Y}\times\mathbf{Z})=(\nabla\times\mathbf{Y})\cdot\mathbf{Z}-\mathbf{Y}\cdot(\nabla\times\mathbf{Z})$ (1)

Substituting (Y = a and Z = A), (Y = a and Z = E), and (Y = a and Z = B), respectively, Equation (1) gives New field equations:

$\nabla\cdot(\mathbf{A}\times\mathbf{A})=(\nabla\times\mathbf{A})\cdot\mathbf{A}-\mathbf{A}\cdot(\nabla\times\mathbf{A})=-\mathbf{A}\cdot\mathbf{B}$. (3.58)

$\nabla\cdot(\mathbf{A}\times\mathrm{E})=(\nabla\times\mathbf{A})\cdot\mathrm{E}-\mathbf{A}\cdot(\nabla\times\mathrm{E})=\mathbf{A}\cdot(\partial\mathbf{B}/\partial\mathbf{t})$. (3.59)

$\nabla\cdot(\mathbf{A}\times\mathbf{B})=(\nabla\times\mathbf{A})\cdot\mathbf{B}-\mathbf{A}\cdot(\nabla\times\mathbf{B})=-\mathbf{A}\cdot\mathrm{j}-\mathbf{A}\cdot(\partial\mathrm{E}/\partial\mathbf{t})$. (3.69)

Curl of cross product of acceleration a and vector fields: $\nabla\times(\mathbf{A}\times\mathbf{A})$, $\nabla\times(\mathbf{A}\times E)$, And $\nabla\times(\mathbf{A}\times\mathbf{B})$

$$\nabla\times(\mathbf{Y}\times\mathbf{Z})=\mathbf{Y}(\nabla\cdot\mathbf{Z})-\mathbf{Z}(\nabla\cdot\mathbf{Y})+(\mathbf{Z}\cdot\nabla)\mathbf{Y}-(\mathbf{Y}\cdot\nabla)\mathbf{Z} \quad (2)$$

Substituting (Y = a and Z = A), (Y = a and Z = E), and (Y = a and Z = B), respectively, Equation (2) gives NEW field equations:

$$\nabla\times(\mathbf{A}\times\mathbf{A})=\mathbf{A}(\nabla\cdot\mathbf{A})-\mathbf{A}(\nabla\cdot\mathbf{A})+(\mathbf{A}\cdot\nabla)\mathbf{A}-(\mathbf{A}\cdot\nabla)\mathbf{A}$$
$$=\mathbf{A}(\nabla\cdot\mathbf{A})-(\mathbf{A}\cdot\nabla)\mathbf{A}. \quad (3.61)$$

$$\nabla\times(\mathbf{A}\times E)=\mathbf{A}(\nabla\cdot E)-E(\nabla\cdot\mathbf{A})+(E\cdot\nabla)\mathbf{A}-(\mathbf{A}\cdot\nabla)E$$
$$=q\mathbf{A}-(\mathbf{A}\cdot\nabla)E. \quad (3.62)$$

$$\nabla\times(\mathbf{A}\times\mathbf{B})=\mathbf{A}(\nabla\cdot\mathbf{B})-\mathbf{B}(\nabla\cdot\mathbf{A})+(\mathbf{B}\cdot\nabla)\mathbf{A}-(\mathbf{A}\cdot\nabla)\mathbf{B}$$
$$=-(\mathbf{A}\cdot\nabla)\mathbf{B}. \quad (3.63)$$

Divergence of product of acceleration a and scalar potential φ: $\nabla\cdot(\varphi\mathbf{A})$

$$\nabla\cdot(\varphi\mathbf{Y})=\varphi(\nabla\cdot\mathbf{Y})+\mathbf{Y}\cdot(\nabla\varphi) \quad (7)$$

Substituting Y = a into Equation (7), we obtain NEW field equation:

$$\nabla\cdot(\varphi\mathbf{A})=\varphi(\nabla\cdot\mathbf{A})+\mathbf{A}\cdot(\nabla\varphi)=\mathbf{A}\cdot(\nabla\varphi) \quad (3.64)$$

Curl of product of acceleration a and scalar field φ: $\nabla\times(\varphi\mathbf{A})$

$$\nabla\times(\varphi\mathbf{Y})=\varphi(\nabla\times\mathbf{Y})+(\nabla\varphi)\times\mathbf{Y}. \quad (8)$$

Substituting Y = a into Equation (8), we obtain NEW field equation:

$$\nabla\times(\varphi\mathbf{A})=\varphi(\nabla\times\mathbf{A})+(\nabla\varphi)\times\mathbf{A}=(\nabla\varphi)\times\mathbf{A} \quad (3.65)$$

Gradient of scalar product of acceleration a and vector fields: $\nabla(\mathbf{A}\cdot\mathbf{A})$, $\nabla(\mathbf{A}\cdot E)$, And $\nabla(\mathbf{A}\cdot\mathbf{B})$

$$\nabla(\mathbf{Y}\cdot\mathbf{Z})=(\mathbf{Y}\cdot\nabla)\mathbf{Z}+(\mathbf{Z}\cdot\nabla)\mathbf{Y}+\mathbf{Y}\times(\nabla\times\mathbf{Z})+\mathbf{Z}\times(\nabla\times\mathbf{Y}). \quad (9)$$

Substituting (Y = a, Z = A), (Y = a, Z = E), and (Y = a and Z = B), into Equation (9), respectively, we obtain NEW field equations:

$$\nabla(\mathbf{A}\cdot\mathbf{A})=(\mathbf{A}\cdot\nabla)\mathbf{A}+(\mathbf{A}\cdot\nabla)\mathbf{A}+\mathbf{A}\times(\nabla\times\mathbf{A})+\mathbf{A}\times(\nabla\times\mathbf{A})$$
$$=(\mathbf{A}\cdot\nabla)\mathbf{A}+\mathbf{A}\times\mathbf{B} \quad (3.66)$$

$$\nabla(\mathbf{A}\cdot E)=(\mathbf{A}\cdot\nabla)E+(E\cdot\nabla)\mathbf{A}+\mathbf{A}\times(\nabla\times E)+E\times(\nabla\times\mathbf{A})$$
$$=(\mathbf{A}\cdot\nabla)E+\mathbf{A}\times(\nabla\times E). \quad (5.10)$$

$$\nabla(\mathbf{A}\cdot\mathbf{B})=(\mathbf{A}\cdot\nabla)\mathbf{B}+(\mathbf{B}\cdot\nabla)\mathbf{A}+\mathbf{A}\times(\nabla\times\mathbf{B})+\mathbf{B}\times(\nabla\times\mathbf{A})$$
$$=(\mathbf{A}\cdot\nabla)\mathbf{B}+\mathbf{A}\times(\nabla\times\mathbf{B}). \quad (3.67)$$

Cross Product: $(\mathbf{A}\times\nabla)\cdot\mathbf{A}$, $(\mathbf{A}\times\nabla)\cdot E$, And $(\mathbf{A}\times\nabla)\cdot\mathbf{B}$

$$(\mathbf{Y}\times\nabla)\cdot\mathbf{Z}=\mathbf{Y}\cdot(\nabla\times\mathbf{Z}). \quad (10)$$

Substituting (Y=a and Z=A), (Y=a and Z=E), and (Y=a and Z=B), respectively, Equation (10) gives new field equations,

$$(\mathbf{A}\times\nabla)\cdot\mathbf{A}=\mathbf{A}\cdot(\nabla\times\mathbf{A})=\mathbf{A}\cdot\mathbf{B}. \quad (3.68)$$

$$(\mathbf{A}\times\nabla)\cdot E=\mathbf{A}\cdot(\nabla\times E)=-\mathbf{A}\cdot\partial(\mathbf{B})/\partial\mathbf{t}. \quad (3.69)$$

$$(\mathbf{A}\times\nabla)\cdot\mathbf{B}=\mathbf{A}\cdot(\nabla\times\mathbf{B})=\mathbf{A}\cdot j+\mathbf{A}\cdot\partial(E)/\partial\mathbf{t}. \quad (3.70)$$

Cross Product: $(\mathbf{A}\times\nabla)\times\mathbf{A}$, $(\mathbf{A}\times\nabla)\times E$, And $(\mathbf{A}\times\nabla)\times\mathbf{B}$

$$(\mathbf{Y}\times\nabla)\times\mathbf{Z}=\mathbf{Y}\times(\nabla\times\mathbf{Z})+(\mathbf{Y}\cdot\nabla)\mathbf{Z}-\mathbf{Y}(\nabla\cdot\mathbf{Z}) \quad (11)$$

Substituting (Y=a and Z=A), (Y=a and Z=E), and (Y=a and Z=B), respectively, Equation (11) gives new field equations:

$$(\mathbf{A}\times\nabla)\times\mathbf{A}=\mathbf{A}\times(\nabla\times\mathbf{A})+(\mathbf{A}\cdot\nabla)\mathbf{A}-\mathbf{A}(\nabla\cdot\mathbf{A})$$
$$=\mathbf{A}\times\mathbf{B}+(\mathbf{A}\cdot\nabla)\mathbf{A}-\mathbf{A}(\nabla\cdot\mathbf{A}). \quad (3.71)$$

$$(\mathbf{A}\times\nabla)\times E=\mathbf{A}\times(\nabla\times E)+(\mathbf{A}\cdot\nabla)E-\mathbf{A}(\nabla\cdot E)$$
$$=\mathbf{A}\times(\nabla\times E)+(\mathbf{A}\cdot\nabla)E-\rho\mathbf{A}. \quad (3.72)$$

$$(\mathbf{A}\times\nabla)\times\mathbf{B}=\mathbf{A}\times(\nabla\times\mathbf{B})+(\mathbf{A}\cdot\nabla)\mathbf{B}-\mathbf{A}(\nabla\cdot\mathbf{B})$$
$$=\mathbf{A}\times(\nabla\times\mathbf{B})+(\mathbf{A}\cdot\nabla)\mathbf{B}. \quad (3.73)$$

Electric field E and magnetic field B created by spin S of electric charge

Divergence of cross product of Spin and vector field: $\nabla\cdot(\mathbf{S}\times\mathbf{A})$, $\nabla\cdot(\mathbf{S}\times E)$, And $\nabla\cdot(\mathbf{S}\times\mathbf{B})$

$$\nabla\cdot(\mathbf{Y}\times\mathbf{Z})=(\nabla\times\mathbf{Y})\cdot\mathbf{Z}-\mathbf{Y}\cdot(\nabla\times\mathbf{Z}) \quad (1)$$

Substituting (Y = S (Spin), Z = A), (Y = S, Z = E), (Y = S, Z = B), respectively, Equation (1) gives New field equation

$$\nabla\cdot(\mathbf{S}\times\mathbf{A})=(\nabla\times\mathbf{S})\cdot\mathbf{A}-\mathbf{S}\cdot(\nabla\times\mathbf{A})=-\mathbf{S}\cdot\mathbf{B}. \quad (3.74)$$

$$\nabla\cdot(\mathbf{S}\times E)=(\nabla\times\mathbf{S})\cdot E-\mathbf{S}\cdot(\nabla\times E)=\mathbf{S}\cdot\partial(\mathbf{B})/\partial\mathbf{t}. \quad (3.75)$$

$$\nabla\cdot(\mathbf{S}\times\mathbf{B})=(\nabla\times\mathbf{S})\cdot\mathbf{B}-\mathbf{S}\cdot(\nabla\times\mathbf{B})=-\mathbf{S}\cdot j-\mathbf{S}\cdot\partial(E)/\partial\mathbf{t} \quad (3.76)$$

Curl of cross product of Spin and vector fields: $\nabla\times(\mathbf{S}\times\mathbf{A})$, $\nabla\times(\mathbf{S}\times E)$, And $\nabla\times(\mathbf{S}\times\mathbf{B})$

$$\nabla\times(\mathbf{Y}\times\mathbf{Z})=\mathbf{Y}(\nabla\cdot\mathbf{Z})-\mathbf{Z}(\nabla\cdot\mathbf{Y})+(\mathbf{Z}\cdot\nabla)\mathbf{Y}-(\mathbf{Y}\cdot\nabla)\mathbf{Z} \quad (2)$$

Substituting (Y = S and Z = A), (Y = S and Z = E), and (Y = S and Z = B), respectively, Equation (2) gives NEW field equations:

$$\nabla\times(\mathbf{S}\times\mathbf{A})=\mathbf{S}(\nabla\cdot\mathbf{A})-\mathbf{A}(\nabla\cdot\mathbf{S})+(\mathbf{A}\cdot\nabla)\mathbf{S}-(\mathbf{S}\cdot\nabla)\mathbf{A}$$
$$=\mathbf{S}(\nabla\cdot\mathbf{A})-(\mathbf{S}\cdot\nabla)\mathbf{A}. \quad (3.77)$$

$$\nabla\times(\mathbf{S}\times E)=\mathbf{S}(\nabla\cdot E)-E(\nabla\cdot\mathbf{S})+(E\cdot\nabla)\mathbf{S}-(\mathbf{S}\cdot\nabla)E$$
$$=\rho\mathbf{S}-(\mathbf{S}\cdot\nabla)E. \quad (3.78)$$

$$\nabla\times(\mathbf{S}\times\mathbf{B})=\mathbf{S}(\nabla\cdot\mathbf{B})-\mathbf{B}(\nabla\cdot\mathbf{S})+(\mathbf{B}\cdot\nabla)\mathbf{S}-(\mathbf{S}\cdot\nabla)\mathbf{B}$$
$$=-(\mathbf{S}\cdot\nabla)\mathbf{B}. \quad (3.79)$$

Divergence of product of scalar potential φ and Spin S: $\nabla\cdot(\varphi\mathbf{S})$

$$\nabla\cdot(\varphi\mathbf{Y})=\varphi(\nabla\cdot\mathbf{Y})+\mathbf{Y}\cdot(\nabla\varphi) \quad (7)$$

Substituting Y = S (Spin) into Equation (7), we obtain NEW field equations:

$$\nabla\cdot(\varphi\mathbf{S})=\varphi(\nabla\cdot\mathbf{S})+\mathbf{S}\cdot(\nabla\varphi)=\mathbf{S}\cdot(\nabla\varphi) \quad (3.80)$$

Curl of product of scalar field φ and Spin S: $\nabla\times(\varphi\mathbf{S})$

$$\nabla\times(\varphi\mathbf{Y})=\varphi(\nabla\times\mathbf{Y})+(\nabla\varphi)\times\mathbf{Y}. \quad (8)$$

Substituting Y = S into Equation (8), we obtain NEW field equations:

$$\nabla\times(\varphi\mathbf{S})=\varphi(\nabla\times\mathbf{S})+(\nabla\varphi)\times\mathbf{S}=(\nabla\varphi)\times\mathbf{S} \quad (3.81)$$

Gradient of scalar product of Spin and vector fields: $\nabla(\mathbf{S}\cdot\mathbf{A})$, $\nabla(\mathbf{S}\cdot E)$, And $\nabla(\mathbf{S}\cdot\mathbf{B})$

$$\nabla(\mathbf{Y}\cdot\mathbf{Z})=(\mathbf{Y}\cdot\nabla)\mathbf{Z}+(\mathbf{Z}\cdot\nabla)\mathbf{Y}+\mathbf{Y}\times(\nabla\times\mathbf{Z})+\mathbf{Z}\times(\nabla\times\mathbf{Y}). \quad (9)$$

Substituting (Y = S and Z=A), (Y = S and Z=E), and (Y = S and Z=B), into Equation (9), respectively, we obtain NEW field equation:

$$\nabla(\mathbf{S}\cdot\mathbf{A})=(\mathbf{S}\cdot\nabla)\mathbf{A}+(\mathbf{A}\cdot\nabla)\mathbf{S}+\mathbf{S}\times(\nabla\times\mathbf{A})+\mathbf{A}\times(\nabla\times\mathbf{S})$$
$$=(\mathbf{S}\cdot\nabla)\mathbf{A}+\mathbf{S}\times\mathbf{B} \quad (3.82)$$

$$\nabla(\mathbf{S}\cdot E)=(\mathbf{S}\cdot\nabla)E+(E\cdot\nabla)\mathbf{S}+\mathbf{S}\times(\nabla\times E)+E\times(\nabla\times\mathbf{S})$$

$$=(\mathbf{S}\cdot\nabla)E+\mathbf{S}\times(\nabla\times E). \quad (3.83)$$

$$\nabla(\mathbf{S}\cdot\mathbf{B})=(\mathbf{S}\cdot\nabla)\mathbf{B}+(\mathbf{B}\cdot\nabla)\mathbf{S}+\mathbf{S}\times(\nabla\times\mathbf{B})+\mathbf{B}\times(\nabla\times\mathbf{S})$$
$$=(\mathbf{S}\cdot\nabla)\mathbf{B}+\mathbf{S}\times(\nabla\times\mathbf{B}). \quad (3.84)$$

Cross Product: $(\mathbf{S}\times\nabla)\cdot\mathbf{A}$, $(\mathbf{S}\times\nabla)\cdot E$, And $(\mathbf{S}\times\nabla)\cdot\mathbf{B}$

$$(\mathbf{Y}\times\nabla)\cdot\mathbf{Z}=\mathbf{Y}\cdot(\nabla\times\mathbf{Z}). \quad (10)$$

Substituting (Y=S and Z=A), (Y=S and Z=E), and (Y=S and Z=B), respectively, Equation (10) gives new field equation,

$$(\mathbf{S}\times\nabla)\cdot\mathbf{A}=\mathbf{S}\cdot(\nabla\times\mathbf{A})=\mathbf{S}\cdot\mathbf{B}. \quad (3.85)$$
$$(\mathbf{S}\times\nabla)\cdot E=\mathbf{S}\cdot(\nabla\times E)=-\mathbf{S}\cdot\partial(\mathbf{B})/\partial\mathbf{t}. \quad (3.86)$$
$$(\mathbf{S}\times\nabla)\cdot\mathbf{B}=\mathbf{S}\cdot(\nabla\times\mathbf{B})=\mathbf{S}\cdot j+\mathbf{S}\cdot\partial(E)/\partial\mathbf{t}. \quad (3.87)$$

Cross Product: $(\mathbf{S}\times\nabla)\times\mathbf{A}$, $(\mathbf{S}\times\nabla)\times E$, and $(\mathbf{S}\times\nabla)\times\mathbf{B}$

$$(\mathbf{Y}\times\nabla)\times\mathbf{Z}=\mathbf{Y}\times(\nabla\times\mathbf{Z})+(\mathbf{Y}\cdot\nabla)\mathbf{Z}-\mathbf{Y}(\nabla\cdot\mathbf{Z}) \quad (11)$$

Substituting (Y=S and Z=A), (Y=S and Z=E), and (Y=S and Z=B), respectively, Equation (11) gives new field equations:

$$(\mathbf{S}\times\nabla)\times\mathbf{A}=\mathbf{S}\times(\nabla\times\mathbf{A})+(\mathbf{S}\cdot\nabla)\mathbf{A}-\mathbf{S}(\nabla\cdot\mathbf{A})$$
$$=\mathbf{S}\times\mathbf{B}+(\mathbf{S}\cdot\nabla)\mathbf{A}-\mathbf{S}(\nabla\cdot\mathbf{A}). \quad (3.88)$$

$$(\mathbf{S}\times\nabla)\times E=\mathbf{S}\times(\nabla\times E)+(\mathbf{S}\cdot\nabla)E-\mathbf{S}(\nabla\cdot E)$$
$$=\mathbf{S}\times(\nabla\times E)+(\mathbf{S}\cdot\nabla)E-\rho\mathbf{S}. \quad (3.89)$$

$$(\mathbf{S}\times\nabla)\times\mathbf{B}=\mathbf{S}\times(\nabla\times\mathbf{B})+(\mathbf{S}\cdot\nabla)\mathbf{B}-\mathbf{S}(\nabla\cdot\mathbf{B})$$
$$=\mathbf{S}\times(\nabla\times\mathbf{B})+(\mathbf{S}\cdot\nabla)\mathbf{B}. \quad (3.90)$$

Summery

In Section 3, we show that the vector calculus identities are powerful tool in re-deriving existing physical laws, and in discovering novel physical laws. We rederived mathematically the following existing physical laws by the vector calculus identities:

1. Poynting's Theorem;
2. Gauss's Law for Magnetism;
3. Charge density Continuity equation;
4. Wave equations of potentials φ and A, electric field E, magnetic field B;
5. Faraday's Law of Induction;
6. Time changes of energy density of the magnetic field;
7. Time changes of energy density of the electric field;
8. Energy density of the magnetic field.

Also, in Section 3, applying the vector calculus identities, we further mathematically derived/proposed many new field equations of both electric fields and magnetic fields with sources moving at velocity v, acceleration a, and spin S, respectively. The new field equations need to be tested experimentally.

Classical Gravity to Gravitoelectromagnetics

Newton Gravitational field g to Gravitoelectromagnetic field g and B_g

n Section 4, let Y and Z represent either gravitoelectric field g, or gravitomagnetic field B_g, or vector gravitation potential A_g, respectively, and φ_g is scalar gravitation potential in Equations (5) to Equation (8). Then substituting g, B_g, A_g, and φ_g into vector calculus identities, we obtain the field equations, respectively.

Newton Gravitational field, g, is created by static gravitational charge density ρ_g (mass density), as shown by Equation (4.1a):

$$\nabla\cdot\mathbf{g}=-\rho_g. \quad (4.1\mathbf{A})$$
$$\nabla\cdot E=\rho\_e. \quad (4.1b)$$

Equation (4.1b) is the Coulomb law of the electric field, which has the exactly same form as that of Equation (4.1a). This similarity between the two suggests us to study the effects of a moving gravitational charge density ρ_g v.

Substituting (Y=v, Z=g) into Equation (2) of Vector Calculus Identity,

$$\nabla\times(\mathbf{Y}\times\mathbf{Z})=\mathbf{Y}(\nabla\cdot\mathbf{Z})-\mathbf{Z}(\nabla\cdot\mathbf{Y})+(\mathbf{Z}\cdot\nabla)\mathbf{Y}-(\mathbf{Y}\cdot\nabla)\mathbf{Z} \quad (2)$$

we obtain

$$\nabla\times(v\times\mathbf{g})=-\rho_g\, v-\partial\mathbf{g}/\partial\mathbf{t}. \quad (4.2)$$

Comparing Equation (4.2) with Ampere-Maxwell law (Equation (4.3),

$$\nabla\times\mathbf{B}=\rho\_e\, v+\partial E/\partial\mathbf{t}, \quad (4.3)$$

we propose that Equation (4.2) shows that the mass current ρ_g v creates the gravitomagnetic field B_g, where the gravitomagnetic field, B_g, is defined as,

$$B_g\equiv-v\times g. \quad (4.4)$$

Then, Equation (4.2) becomes

$$\nabla\times\mathbf{B_g}=J_g+\partial\mathbf{g}/\partial\mathbf{t}. \quad (4.5)$$
$$J_g=\rho_g\, v.$$

Following the electromagnetics, let define the gravitomagnetic field B_g in term of the gravitation vector potential A_g, and define the gravitational field g in terms of the gravitational vector potential A_g and the gravitation scalar potential φ_g,

$$\mathbf{B_g}\equiv\nabla\times\mathbf{A_g}, \quad (4.6)$$
$$\mathbf{g}\equiv-\nabla\varphi_g-\partial\mathbf{A_g}/\partial\mathbf{t}. \quad (4.7)$$

Equation (4.6) and Equation (4.7) give,

$$\nabla\cdot\mathbf{B_g}=0, \quad (4.8)$$
$$\nabla\times\mathbf{g}=-\partial\mathbf{B_g}/\partial\mathbf{t} \quad (4.9)$$

Equation (4.1a), Equation (4.5), Equation (4.8) and Equation (4.9) are Maxwell-type equations for gravity field.

Note: by introducing the definition of the g field, Equation (4.7), Equation (4.1a) leads to the wave Equation (4.1c) of the scaler gravitation potential φ_g,

$$\nabla\cdot\mathbf{g}=-\nabla^2\varphi_g+(\partial^2\varphi_g)/(\partial\mathbf{t}^2)=-\rho_g. \quad (4.1c)$$

Next, we study the gravitoelectric field g and gravitomagnetic field B_g further in the same way of studying the electric field E and magnetic field B in Section 3.

Divergence of cross product of vector gravitation fields: $\nabla\cdot(\mathbf{g}\times\mathbf{B_g})$, $\nabla\cdot(\mathbf{A_g}\times\mathbf{B_g})$, $\nabla\cdot(\mathbf{A_g}\times\mathbf{g})$

$$\nabla\cdot(\mathbf{Y}\times\mathbf{Z})=(\nabla\times\mathbf{Y})\cdot\mathbf{Z}-\mathbf{Y}\cdot(\nabla\times\mathbf{Z}) \quad (1)$$

Poynting-type Gravitation Theorem

Substituting (Y = g, Z = B_g), Equation (1) gives Poynting-type Gravitation Theorem:

$$\nabla\cdot(\mathbf{g}\times\mathbf{B_g})=-\mathbf{g}\cdot\mathbf{j_g}-1/2\,\partial(\mathbf{g}^2+[\mathbf{B_g}]^2)/\partial\mathbf{t} \quad (4.10)$$

Where $\nabla\times B_g=j_g+(\partial g/\partial t)$ and $j_g=\rho_g$ v.

New gravitation field equation

Substituting (Y = A_g, Z = B_g) and (Y = A_g, Z = g), respectively, Equation (1) gives NEW gravitation field equation:

$$\nabla\cdot(\mathbf{A_g}\times\mathbf{B_g})=\mathbf{B_g}{}^2-\mathbf{A_g}\cdot\mathbf{j_g}-\mathbf{A_g}\cdot(\partial\mathbf{g}/\partial\mathbf{t}). \quad (4.11)$$

$$\nabla\cdot(\mathbf{A_g}\times\mathbf{g})=\mathbf{g}\cdot\mathbf{B_g}+\mathbf{A_g}\cdot(\partial\mathbf{B_g}/\partial\mathbf{t}).. \quad (4.12)$$

Equation (4.11) represent the energy of magnetic field. In electromagnetics, the term "E·B" is the Lorentz invariant. Similarly, the term, "g·B_g", of Equation (4.12) is Lorentz scalar invariant.

Curl of cross product of two vector gravitation fields: $\nabla\times(\mathbf{g}\times\mathbf{B}_g)$, $\nabla\times(\mathbf{A}_g\times\mathbf{B}_g)$, **And** $\nabla\times(\mathbf{A}_g\times\mathbf{g})$

$$\nabla\times(\mathbf{Y}\times\mathbf{Z})=\mathbf{Y}(\nabla\cdot\mathbf{Z})-\mathbf{Z}(\nabla\cdot\mathbf{Y})+(\mathbf{Z}\cdot\nabla)\mathbf{Y}-(\mathbf{Y}\cdot\nabla)\mathbf{Z} \tag{2}$$

Substituting ($Y = g$, $Z = B_g$), ($Y = A_g$, $Z = B_g$), and ($Y = A_g$, $Z = g$), respectively, Equation (2) gives

NEW gravitation field equations:

$$\nabla\times(\mathbf{g}\times\mathbf{B}_g)=\rho_g\,\mathbf{B}_g+(\mathbf{B}_g\cdot\nabla)\mathbf{g}-(\mathbf{g}\cdot\nabla)\,\mathbf{B}_g \tag{4.13}$$

$$\nabla\times(\mathbf{A}_g\times\mathbf{B}_g)=-\mathbf{B}_g\,(\nabla\cdot\mathbf{A}_g)+(\mathbf{B}_g\cdot\nabla)\,\mathbf{A}_g-(\mathbf{A}_g\cdot\nabla)\,\mathbf{B}_g \tag{4.14}$$

$$\nabla\times(\mathbf{A}_g\times\mathbf{g})=-\rho_g\,\mathbf{A}_g-\mathbf{g}(\nabla\cdot\mathbf{A}_g)+(\mathbf{g}\cdot\nabla)\,\mathbf{A}_g-(\mathbf{A}_g\cdot\nabla)\mathbf{g}. \tag{4.15}$$

Divergence of Curl of vector gravitation field: $\nabla\times\mathbf{Y}$

$$\nabla\cdot(\nabla\times\mathbf{Y})=0 \tag{3}$$

Gauss-type Law for Gravitomagnetics (1)

Substituting $Y = A_g$, Equation (3) gives Gauss-type Law for gravitomagnetics:

$$\nabla\cdot(\nabla\times\mathbf{A}_g)=\nabla\cdot\mathbf{B}_g=0 \tag{4.16}$$

Equation (4.16) mathematically indicates no gravitomagnetic monopoles.

Gauss-type Law for gravitomagnetics (2)

Substituting $Y = g$, Equation (3) gives:

$$\nabla\cdot(\nabla\times\mathbf{g})=0 \tag{4.17}$$

Substituting Equation (4.9) into Equation (4.17), we obtain:

$$\nabla\cdot(\nabla\times\mathbf{g})=\nabla\cdot(-\partial\mathbf{B}_g/\partial\mathbf{t})=-\partial/\partial\mathbf{t}\,(\nabla\cdot\mathbf{B}_g)=0. \tag{4.18}$$

Equation (4.18) has two solutions:

1. The gravitomagnetic field B_g is a constant field:

$$\partial B_g/\partial t=0. \tag{4.19}$$

2. Gauss-type Law for gravitomagnetics

$$\nabla\cdot\mathbf{B}_g\,(\mathbf{t})=0. \tag{4.20}$$

Gravitition-Charge Density Continuity Equation

Substituting $Y = B_g$, Equation (3) gives:

$$\nabla\cdot(\nabla\times\mathbf{B}_g)=0 \tag{4.21}$$

Substituting Equation (4.5) into Equation (4.21), we obtain the Gravitation-charge Density Continuity Equation,

$$\nabla\cdot(\nabla\times\mathbf{B}_g)=\nabla\cdot J_g-\partial/\partial t\,\rho_g=0 \tag{4.22}$$

Curl of curl of vector gravitation field: wave equation

$$\nabla\times(\nabla\times\mathbf{Y})=\nabla(\nabla\cdot\mathbf{Y})-\nabla^2\,\mathbf{Y} \tag{4}$$

Wave equation of vector gravitation potential A_g

Substituting $Y = A_g$, Equation (4) gives:

$$\nabla\times(\nabla\times\mathbf{A}_g)=j_g+\partial/\partial\mathbf{t}\,\mathbf{g} \tag{4.23A}$$

and

$$\nabla(\nabla\cdot\mathbf{A}_g)-\nabla^2\,\mathbf{A}_g=\partial/\partial\mathbf{t}\,\mathbf{g}+\partial^2/(\partial\mathbf{t}^2)\,\mathbf{A}_g-\nabla^2\,\mathbf{A}_g \tag{4.23B}$$

Combining Equation (4.23a) and Equation (4.23b), we obtain wave equation of vector potential A_g:

$$\partial^2/(\partial\mathbf{t}^2)\,\mathbf{A}_g-\nabla^2\,\mathbf{A}_g=j_g \tag{4.23c}$$

Wave equation of gravitoelectric field g

Substituting $Y = g$, Equation (4) gives

$$\nabla\times(\nabla\times\mathbf{g})=-\partial/\partial\mathbf{t}\,j_g-\partial^2/(\partial\mathbf{t}^2)\,\mathbf{g} \tag{4.24A}$$

And

$$\nabla(\nabla\cdot\mathbf{g})-\nabla^2\,\mathbf{g}=\nabla\rho_g-\nabla^2\,\mathbf{g} \tag{4.24b}$$

Combining Equation (4.24a) and Equation (4.24b), we obtain wave equation of gravitoelectric field g,

$$\partial^2/(\partial\mathbf{t}^2)\,\mathbf{g}-\nabla^2\,\mathbf{g}=-\partial/\partial\mathbf{t}\,j_g-\nabla\rho_g \tag{4.24c}$$

Wave equation of gravitomagnetic field B_g

Substituting $Y = B_g$, Equation (4) gives

$$\nabla\times(\nabla\times\mathbf{B}_g)=\nabla\times j_g-\partial^2/(\partial\mathbf{t}^2)\,\mathbf{B}_g \tag{4.25A}$$

And

$$\nabla(\nabla\cdot\mathbf{B}_g)-\nabla^2\,\mathbf{B}_g=-\nabla^2\,\mathbf{B}_g \tag{4.25b}$$

Combining Equation (4.25a) and Equation (4.25b), we obtain wave equation of B_g,

$$\partial^2/(\partial\mathbf{t}^2)\,\mathbf{B}_g-\nabla^2\,\mathbf{B}_g=\nabla\times j_g \tag{4.25c}$$

Divergence of Gradient of scalar field: wave equation of gravitation scalar potential φ_g

$$\nabla\cdot(\nabla\varphi)=\nabla^2\,\varphi \tag{5}$$

Substituting the definition of gravitoelectric field, $\mathbf{g}=-\nabla\varphi_g-\partial/\partial t\,A_g$ into Equation (5). We obtain wave equation of gravitation scalar potential φ_g:

$$\nabla\cdot(\nabla\varphi_g)=\rho_g+\partial^2/(\partial\mathbf{t}^2)\,\varphi_g=\nabla^2\,\varphi_g \tag{4.26A}$$

i.e.,

$$\partial^2/(\partial\mathbf{t}^2)\,\varphi_g-\nabla^2\,\varphi_g=-\rho_g. \tag{4.26b}$$

Curl of Gradient of gravitation scalar field: Faraday-type gravitation Law of Induction

$$\nabla\times(\nabla\varphi)=0 \tag{6}$$

Substituting the definition of the gravitoelectric field, $g\equiv-\nabla\varphi_g-\partial\mathbf{A}_g/\partial\mathbf{t}$, into Equation (6).

We obtain Faraday-type gravitation Law of Induction:

$$\nabla\times(\nabla\varphi_g)=-\nabla\times\mathbf{g}-\partial\mathbf{B}_g/\partial\mathbf{t}=0. \tag{4.27a}$$

i.e.,

$$\nabla\times\mathbf{g}=-\partial\mathbf{B}_g/\partial\mathbf{t}. \tag{4.27b}$$

Divergence of product of gravitation scalar potential φ_g with gravitation vector potential A_g, gravitoelectric field g, gravitomagnetic field B_g, respectively

$$\nabla\cdot(\varphi\mathbf{Y})=\varphi(\nabla\cdot\mathbf{Y})+\mathbf{Y}\cdot(\nabla\varphi) \tag{7}$$

Substituting $Y = A_g$, $Y = g$, and $Y = B_g$ into Equation (7), respectively, we obtain NEW gravitation field equations:

$$\nabla\cdot(\varphi_g\,\mathbf{A}_g)=\varphi_g\,(\nabla\cdot\mathbf{A}_g)+\mathbf{A}_g\cdot(\nabla\varphi_g) \tag{4.28}$$

$$\nabla\cdot(\varphi_g\,\mathbf{g})=-\rho_g\,\varphi_g+\mathbf{g}\cdot(\nabla\varphi_g) \tag{4.29}$$

$$\nabla\cdot(\varphi_g\,\mathbf{B}_g)=\mathbf{B}_g\cdot(\nabla\varphi_g) \tag{4.30}$$

Curl of product of gravitation scalar potential φ_g and vector gravity field*:* A_g, g, B_g

$$\nabla\times(\varphi\mathbf{Y})=\varphi(\nabla\times\mathbf{Y})+(\nabla\varphi)\times\mathbf{Y}. \tag{8}$$

Substituting $Y = A_g$, $Y = g$, and $Y = B_g$ into Equation (8), respectively, we obtain NEW gravitation field equations:

$$\nabla\times(\varphi_g\,\mathbf{A}_g)=\varphi_g\,\mathbf{B}_g+(\nabla\varphi_g)\times\mathbf{A}_g \tag{4.31}$$

$$\nabla\times(\varphi_g\,\mathbf{g})=-\varphi_g\,\partial\mathbf{B}_g/\partial\mathbf{t}+(\nabla\varphi_g)\times\mathbf{g} \tag{4.32}$$

$$\nabla\times(\varphi_g\,\mathbf{B}_g)=J_g\,\varphi_g+\varphi_g\,\partial\mathbf{g}/\partial\mathbf{t}+(\nabla\varphi_g)\times\mathbf{B}_g \tag{4.33}$$

Gradient of scalar product of two gravitation vector fields: $\nabla(\mathbf{g}\cdot\mathbf{B}_g)$, $\nabla(\mathbf{A}_g\cdot\mathbf{B}_g)$, **And** $\nabla(\mathbf{A}_g\cdot\mathbf{g})$

$$\nabla(\mathbf{Y}\cdot\mathbf{Z})=(\mathbf{Y}\cdot\nabla)\mathbf{Z}+(\mathbf{Z}\cdot\nabla)\mathbf{Y}+\mathbf{Y}\times(\nabla\times\mathbf{Z})+\mathbf{Z}\times(\nabla\times\mathbf{Y}). \tag{9}$$

Substituting ($Y = g$, $Z = B_g$), ($Y = A_g$, $Z=B_g$), ($Y=A_g$, $Z=g$), ($Y = Z = B_g$), ($Y = Z =A_g$), ($Y=Z=g$), into Equation (9), respectively, we obtain NEW gravitation field equations:

$$\nabla(\mathbf{g}\cdot\mathbf{B}_g)=(\mathbf{g}\cdot\nabla)\,\mathbf{B}_g+(\mathbf{B}_g\cdot\nabla)\mathbf{g}+\mathbf{g}\times J_g \tag{4.34}$$

$$\nabla(\mathbf{A}_g\cdot\mathbf{B}_g)=(\mathbf{A}_g\cdot\nabla)\,\mathbf{B}_g+(\mathbf{B}_g\cdot\nabla)\,\mathbf{A}_g+\mathbf{A}_g\times J_g+\mathbf{A}_g\times\partial\mathbf{g}/\partial\mathbf{t} \tag{4.35}$$

$$\nabla(\mathbf{A}_g\cdot\mathbf{g})=(\mathbf{A}_g\cdot\nabla)\mathbf{g}+(\mathbf{g}\cdot\nabla)\,\mathbf{A}_g-\mathbf{A}_g\times\partial\mathbf{B}_g/\partial\mathbf{t}+\mathbf{g}\times\mathbf{B}_g \tag{4.36}$$

$$\nabla(\mathbf{A}_g\cdot\mathbf{A}_g)=2(\mathbf{A}_g\cdot\nabla)\,\mathbf{A}_g+2\mathbf{A}_g\times\mathbf{B}_g \tag{4.37}$$

$$\nabla(\mathbf{g}\cdot\mathbf{g})=2(\mathbf{g}\cdot\nabla)\mathbf{g}-2\mathbf{g}\times(\partial\mathbf{B}_g/\partial\mathbf{t}) \tag{4.38}$$

$$\nabla(\mathbf{B}_g\cdot\mathbf{B}_g)=2(\mathbf{B}_g\cdot\nabla)\,\mathbf{B}_g+2\mathbf{B}_g\times(\nabla\times\mathbf{B}_g). \tag{4.39}$$

Cross Product: $(\mathbf{Y}\times\nabla)\cdot\mathbf{Z}$

Equation (10),

$$(\mathbf{Y}\times\nabla)\cdot\mathbf{Z}=\mathbf{Y}\cdot(\nabla\times\mathbf{Z}), \tag{10}$$

gives the gravitoelectromagnetics field equations.

Substituting, respectively, ($Y=g$, $Z=B_g$), ($Y=B_g$, $Z=g$),

($Y=A_g$, $Z=B_g$), ($Y=B_g$, $Z=A_g$), ($Y=g$, $Z=A_g$), and ($Y=A_g$, $Z=g$), Equation (10) gives new gravitation field equations:

$(\mathbf{g}\times\nabla)\cdot\mathbf{B}_g=\mathbf{g}\cdot J_g+\mathbf{g}\cdot\partial\mathbf{g}/\partial\mathbf{t}$ (4.40)

$(\mathbf{B}_g\times\nabla)\cdot\mathbf{g}=-\mathbf{B}_g\cdot\partial\mathbf{B}_g/\partial\mathbf{t}.$ (4.41)

$(\mathbf{A}_g\times\nabla)\cdot\mathbf{B}_g=\mathbf{A}_g\cdot J_g+\mathbf{A}_g\cdot\partial\mathbf{g}/\partial\mathbf{t}$ (4.42)

$(\mathbf{B}_g\times\nabla)\cdot\mathbf{A}_g=\mathbf{B}_g\hat{}2$ (4.43)

$(\mathbf{g}\times\nabla)\cdot\mathbf{A}_g=\mathbf{g}\cdot\mathbf{B}_g$ (4.44)

$(\mathbf{A}_g\times\nabla)\cdot\mathbf{g}=-\mathbf{A}_g\cdot\partial\mathbf{B}_g/\partial\mathbf{t}$ (4.45)

Cross Product: $(\mathbf{Y}\times\nabla)\times\mathbf{Z}$

$(\mathbf{Y}\times\nabla)\times\mathbf{Z}=\mathbf{Y}\times(\nabla\times\mathbf{Z})+(\mathbf{Y}\cdot\nabla)\mathbf{Z}-\mathbf{Y}(\nabla\cdot\mathbf{Z})$ (11)

Substituting ($Y=g$, $Z=B_g$), ($Y=B_g$, $Z=g$), ($Y=A_g$, $Z=B_g$), ($Y=B_g$, $Z=A_g$), ($Y=A_g$, $Z=g$), and ($Y=g$, $Z=A_g$), Equation (11) gives new gravitation field equations:

$(\mathbf{g}\times\nabla)\times\mathbf{B}_g=\mathbf{g}\times J_g+(\mathbf{g}\cdot\nabla)\,\mathbf{B}_g$ (4.46)

$(\mathbf{B}_g\times\nabla)\times\mathbf{g}=-\rho\mathbf{B}_g+(\mathbf{B}_g\cdot\nabla)\mathbf{g}$ (4.47)

$(\mathbf{A}_g\times\nabla)\times\mathbf{B}_g=\mathbf{A}_g\times J_g+\mathbf{A}_g\times\partial\mathbf{g}/\partial\mathbf{t}+(\mathbf{A}_g\cdot\nabla)\,\mathbf{B}_g$ (4.48)

$(\mathbf{B}_g\times\nabla)\times\mathbf{A}_g=(\mathbf{B}_g\cdot\nabla)\,\mathbf{A}_g+\mathbf{B}_g\,(\partial\varphi_g/\partial\mathbf{t})$ (4.49)

$(\mathbf{A}_g\times\nabla)\times\mathbf{g}=-\mathbf{A}_g\times\partial\mathbf{B}_g/\partial\mathbf{t}+(\mathbf{A}_g\cdot\nabla)\mathbf{g}-\rho\mathbf{A}_g$ (4.50)

$(\mathbf{g}\times\nabla)\times\mathbf{A}_g=\mathbf{g}\times\mathbf{B}_g+(\mathbf{B}_g\cdot\nabla)\,\mathbf{A}_g+\mathbf{g}(\partial\varphi_g/\partial\mathbf{t})$ (4.51)

In Section 4.1, we have mathematically derived physics laws which have the same forms of electromagnetic laws:

(a) Poynting-type gravity Theorem

(b) Gauss-type Law for gravitomagnetism

(c) Gravitition-Charge density continuity equation

(d) Faraday-type gravitation Law of Induction

(e) Wave equation of gravitation vector potential A_g, gravitoelectric field g and gravitomagnetic field B_g

(f) Energy density of time change of the gravitoelectric field

(g) Energy density of time change of the gravitomagnetic field

(h) Energy density of the gravitomagnetic field

We also derived several new physical laws of gravity.

Gravitoelectric field and Gravitomagnetic field created by velocity v of gravitation mass-charge

We study how Newton gravity field g of rest mass goes to Gravitomagnetic field B_g of moving mass

Divergence of cross product of velocity v and gravitation vector fields, A_g, g, B_g

$\nabla\cdot(\mathbf{Y}\times\mathbf{Z})=(\nabla\times\mathbf{Y})\cdot\mathbf{Z}-\mathbf{Y}\cdot(\nabla\times\mathbf{Z})$ (1)

Substituting ($Y = v$, $Z = A_g$), ($Y = v$, $Z = g$), and ($Y = v$, $Z = B_g$), respectively, Equation (1) gives New gravitation field equations:

$\nabla\cdot(v\times\mathbf{A}_g)=-v\cdot\mathbf{B}_g.$ (4.52)

$\nabla\cdot(v\times\mathbf{g})=v\cdot\partial\mathbf{B}_g/\partial\mathbf{t}.$ (4.53)

$\nabla\cdot(v\times\mathbf{B}_g)=-v\cdot J_g-v\cdot\partial\mathbf{g}/\partial\mathbf{t}$ (4.54)

Curl of cross product of velocity v and gravitation vector fields: A_g, g, B_g

$\nabla\times(\mathbf{Y}\times\mathbf{Z})=\mathbf{Y}(\nabla\cdot\mathbf{Z})-\mathbf{Z}(\nabla\cdot\mathbf{Y})+(\mathbf{Z}\cdot\nabla)\mathbf{Y}-(\mathbf{Y}\cdot\nabla)\mathbf{Z}$ (2)

Substituting ($Y = v$, $Z = A_g$), ($Y = v$, $Z = g$), and ($Y = v$, $Z = B_g$), respectively, Equation (2) gives Maxwell-type gravitation field equations:

$\nabla\times(v\times\mathbf{A}_g)=v(\nabla\cdot\mathbf{A}_g)-\partial\mathbf{A}_g/\partial\mathbf{t}.$ (4.55)

$\nabla\times(v\times\mathbf{g})=v(\nabla\cdot\mathbf{g})-\partial\mathbf{g}/\partial\mathbf{t}$ (4.56)

$\nabla\times(v\times\mathbf{B}_g)=v(\nabla\cdot\mathbf{B}_g)-\partial\mathbf{B}_g/\partial\mathbf{t}.$ (4.57)

Equation (4.56) and Equation (4.57) are perfect dual to each other.

Let us introducing the definitions of gravitomagnetic field B_g and gravitoelectric field g:

$B_g\equiv-v\times g.$ (4.58)

$g\equiv v\times B_g$ (4.59)

Then Equation (4.56) and Equation (4.57) can be written as:

$\nabla\times\mathbf{B}_g=\rho_g\,v+\partial\mathbf{g}/\partial\mathbf{t}$ (4.60)

$\nabla\times\mathbf{g}=-\partial\mathbf{B}_g/\partial\mathbf{t}.$ (4.61)

Equation (4.60) and Equation (4.61) have the same form as that of Ampere-Maxwell equation and Faraday equation, respectively:

$\nabla\times\mathbf{B}=\rho\_e\,v+\partial E/\partial\mathbf{t}.,$

$\nabla\times E=-\partial\mathbf{B}/\partial\mathbf{t}.$

Gradient of scalar product of velocity v and gravitation vector fields: $\nabla(\mathbf{Y}\cdot\mathbf{Z})$

$\nabla(\mathbf{Y}\cdot\mathbf{Z})=(\mathbf{Y}\cdot\nabla)\mathbf{Z}+(\mathbf{Z}\cdot\nabla)\mathbf{Y}+\mathbf{Y}\times(\nabla\times\mathbf{Z})+\mathbf{Z}\times(\nabla\times\mathbf{Y}).$ (9)

Substituting ($Y = v$ and $Z=A_g$), or ($Y = v$ and $Z=g$), or ($Y = v$ and $Z=B_g$), into Equation (9), respectively, we obtain NEW gravitation field equations:

$\nabla(v\cdot\mathbf{A}_g)=(v\cdot\nabla)\,\mathbf{A}_g+v\times\mathbf{B}_g$ (4.62)

$\nabla(v\cdot\mathbf{g})=(v\cdot\nabla)\mathbf{g}+v\times(\nabla\times\mathbf{g}).$ (4.63)

$\nabla(v\cdot\mathbf{B}_g)=(v\cdot\nabla)\,\mathbf{B}_g+v\times(\nabla\times\mathbf{B}_g).$ (4.64)

Cross Product: $(\mathbf{Y}\times\nabla)\cdot\mathbf{Z}$

$(\mathbf{Y}\times\nabla)\cdot\mathbf{Z}=\mathbf{Y}\cdot(\nabla\times\mathbf{Z}).$ (10)

Substituting ($Y=v$, $Z=A_g$), or ($Y=v$,$Z=g$), or ($Y=v$,$Z=B_g$), respectively, Equation (10) gives new gravitation field equations,

$(v\times\nabla)\cdot\mathbf{A}_g=v\cdot\mathbf{B}_g.$ (4.65)

$(v\times\nabla)\cdot\mathbf{g}=-v\cdot\partial\mathbf{B}_g/\partial\mathbf{t}.$ (4.66)

$(v\times\nabla)\cdot\mathbf{B}_g=v\cdot J_g+v\cdot\partial\mathbf{g}/\partial\mathbf{t}.$ (4.67)

Cross Product: $(\mathbf{Y}\times\nabla)\times\mathbf{Z}$

$(\mathbf{Y}\times\nabla)\times\mathbf{Z}=\mathbf{Y}\times(\nabla\times\mathbf{Z})+(\mathbf{Y}\cdot\nabla)\mathbf{Z}-\mathbf{Y}(\nabla\cdot\mathbf{Z})$ (11)

Substituting ($Y=v$, $Z=A_g$), or ($Y=v$,$Z=g$), or ($Y=v$,$Z=B_g$), respectively, Equation (11) gives

new gravitation field equations:

$(v\times\nabla)\times\mathbf{A}_g=v\times\mathbf{B}_g+\partial\mathbf{A}_g/\partial\mathbf{t}-v(\nabla\cdot\mathbf{A}_g).$ (4.68)

$(v\times\nabla)\times\mathbf{g}=v\times(\nabla\times\mathbf{g})+\partial\mathbf{g}/\partial\mathbf{t}-\rho_g\,v.$ (4.69)

$(v\times\nabla)\times\mathbf{B}_g=v\times(\nabla\times\mathbf{B}_g)+\partial\mathbf{B}_g/\partial\mathbf{t}.$ (4.70)

Gravitoelectric field and Gravitomagnetic field created by acceleration "a" of gravitational-charge

Divergence of cross product of acceleration "a" and gravitation vector fields: $\nabla\cdot(\mathbf{Y}\times\mathbf{Z})$

$\nabla\cdot(\mathbf{Y}\times\mathbf{Z})=(\nabla\times\mathbf{Y})\cdot\mathbf{Z}-\mathbf{Y}\cdot(\nabla\times\mathbf{Z})$ (1)

Substituting ($Y = a$, $Z = A_g$), ($Y = a$, $Z = g$), and ($Y = a$, $Z = B_g$), respectively, Equation (1) gives New gravitation field equations:

$\nabla\cdot(\mathbf{A}\times\mathbf{A}_g)=-\mathbf{A}\cdot\mathbf{B}_g.$ (4.71)

$\nabla\cdot(\mathbf{A}\times\mathbf{g})=\mathbf{A}\cdot(\partial\mathbf{B}_g/\partial\mathbf{t}).$ (4.72)

$\nabla\cdot(\mathbf{A}\times\mathbf{B}_g)=-\mathbf{A}\cdot J_g-\mathbf{A}\cdot(\partial\mathbf{g}/\partial\mathbf{t}).$ (4.73)

Curl of cross product of acceleration "a" and gravitation vector fields: $\nabla\times(\mathbf{Y}\times\mathbf{Z})$

$\nabla\times(\mathbf{Y}\times\mathbf{Z})=\mathbf{Y}(\nabla\cdot\mathbf{Z})-\mathbf{Z}(\nabla\cdot\mathbf{Y})+(\mathbf{Z}\cdot\nabla)\mathbf{Y}-(\mathbf{Y}\cdot\nabla)\mathbf{Z}$ (2)

Substituting (**$\mathbf{Y = a, Z = A_g}$**), (**$\mathbf{Y = a, Z = g}$**), and (**$\mathbf{Y = a, Z = B_g}$**), respectively, Equation (2) gives NEW gravitation field equations:

$\nabla\times(\mathbf{A}\times\mathbf{A}_g)=\mathbf{A}(\nabla\cdot\mathbf{A}_g)-(\mathbf{A}\cdot\nabla)\,\mathbf{A}_g.$ (4.74)

$\nabla\times(\mathbf{A}\times\mathbf{g})=\rho_g\,\mathbf{A}-(\mathbf{A}\cdot\nabla)\mathbf{g}.$ (4.75)

$$\nabla\times(\mathbf{A}\times\mathbf{B}_g)=-(\mathbf{A}\cdot\nabla)\,\mathbf{B}_g. \quad (4.76)$$

Equation (4.75) shows that an accelerating gravity-charge, $\rho_g a$, creates a gravitomagnetic-type field, $a\times g$.

Gradient of scalar product of acceleration a and gravitation vector fields: $\nabla(Y\cdot Z)$

$$\nabla(\mathbf{Y}\cdot\mathbf{Z})=(\mathbf{Y}\cdot\nabla)\mathbf{Z}+(\mathbf{Z}\cdot\nabla)\mathbf{Y}+\mathbf{Y}\times(\nabla\times\mathbf{Z})+\mathbf{Z}\times(\nabla\times\mathbf{Y}). \quad (9)$$

Substituting ($\mathbf{Y} = \mathbf{a}$, $\mathbf{Z} = \mathbf{A}_g$), ($\mathbf{Y} = \mathbf{a}$, $\mathbf{Z} = \mathbf{g}$), and ($\mathbf{Y} = \mathbf{a}$, $\mathbf{Z} = \mathbf{B}_g$), into Equation (9), respectively, we obtain NEW gravitation field equations:

$$\nabla(\mathbf{A}\cdot\mathbf{A}_g)=(\mathbf{A}\cdot\nabla)\,\mathbf{A}_g+\mathbf{A}\times\mathbf{B}_g. \quad (4.77)$$

$$\nabla(\mathbf{A}\cdot\mathbf{g})=(\mathbf{A}\cdot\nabla)\mathbf{g}+\mathbf{A}\times(\nabla\times\mathbf{g}). \quad (4.78)$$

$$\nabla(\mathbf{A}\cdot\mathbf{B}_g)=(\mathbf{A}\cdot\nabla)\,\mathbf{B}_g+\mathbf{A}\times(\nabla\times\mathbf{B}_g.). \quad (4.79)$$

Cross Product: $(\mathbf{Y}\times\nabla)\cdot\mathbf{Z}$

$$(\mathbf{Y}\times\nabla)\cdot\mathbf{Z}=\mathbf{Y}\cdot(\nabla\times\mathbf{Z}). \quad (10)$$

Substituting ($\mathbf{Y}=\mathbf{a}$, $\mathbf{Z}=\mathbf{A}_g$), ($\mathbf{Y}=\mathbf{a},\mathbf{Z}=\mathbf{g}$), and ($\mathbf{Y}=\mathbf{a},\mathbf{Z}=\mathbf{B}_g$), respectively, Equation (10) gives new gravitation field equations,

$$(\mathbf{A}\times\nabla)\cdot\mathbf{A}_g=\mathbf{A}\cdot\mathbf{B}_g. \quad (4.80)$$

$$(\mathbf{A}\times\nabla)\cdot\mathbf{g}=-\mathbf{A}\cdot\partial(\mathbf{B}_g)/\partial\mathbf{t}. \quad (4.81)$$

$$(\mathbf{A}\times\nabla)\cdot\mathbf{B}_g=\mathbf{A}\cdot\mathbf{J}_g+\mathbf{A}\cdot\partial(\mathbf{g})/\partial\mathbf{t}. \quad (4.82)$$

Cross Product: $(\mathbf{Y}\times\nabla)\times\mathbf{Z}$

$$(\mathbf{Y}\times\nabla)\times\mathbf{Z}=\mathbf{Y}\times(\nabla\times\mathbf{Z})+(\mathbf{Y}\cdot\nabla)\mathbf{Z}-\mathbf{Y}(\nabla\cdot Z) \quad (11)$$

Substituting ($Y=a$, $Z=A_g$), ($Y=a,Z=g$), and ($Y=a,Z=B_g$), respectively, Equation (11) gives new gravitation field equations:

$$(\mathbf{A}\times\nabla)\times\mathbf{A}_g=\mathbf{A}\times\mathbf{B}_g+(\mathbf{A}\cdot\nabla)\,\mathbf{A}_g-\mathbf{A}(\nabla\cdot\mathbf{A}_g). \quad (4.83)$$

$$(\mathbf{A}\times\nabla)\times\mathbf{g}=\mathbf{A}\times(\nabla\times\mathbf{g})+(\mathbf{A}\cdot\nabla)\mathbf{g}-\rho_g\,\mathbf{A}. \quad (4.84)$$

$$(\mathbf{A}\times\nabla)\times\mathbf{B}_g=\mathbf{A}\times(\nabla\times\mathbf{B}_g)+(\mathbf{A}\cdot\nabla)\,\mathbf{B}_g. \quad (4.85)$$

Gravitoelectric field g and gravitomagnetic field B_g created by Spin S of gravity-charge

Divergence of cross product of Spin and gravitation vector field: $\nabla\cdot(\mathbf{S}\times\mathbf{A}_g)$, $\nabla\cdot(\mathbf{S}\times\mathbf{g})$, $\nabla\cdot(\mathbf{S}\times\mathbf{B}_g)$

$$\nabla\cdot(\mathbf{Y}\times\mathbf{Z})=(\nabla\times\mathbf{Y})\cdot\mathbf{Z}-\mathbf{Y}\cdot(\nabla\times\mathbf{Z}) \quad (1)$$

Substituting ($\mathbf{Y} = \mathbf{S}$, $\mathbf{Z} = \mathbf{A}_g$), ($\mathbf{Y} = \mathbf{S}$, $\mathbf{Z} = \mathbf{g}$), and ($\mathbf{Y} = \mathbf{S}$, $\mathbf{Z} = \mathbf{B}_g$), Equation (1) gives New gravitation field equation

$$\nabla\cdot(\mathbf{S}\times\mathbf{A}_g)=-\mathbf{S}\cdot\mathbf{B}_g. \quad (4.86)$$

$$\nabla\cdot(\mathbf{S}\times\mathbf{g})=\mathbf{S}\cdot\partial(\mathbf{B}_g)/\partial\mathbf{t}. \quad (4.87)$$

$$\nabla\cdot(\mathbf{S}\times\mathbf{B}_g)=-\mathbf{S}\cdot\mathbf{J}_g-\mathbf{S}\cdot\partial\mathbf{g}/\partial\mathbf{t} \quad (4.88)$$

Curl of cross product of Spin and gravitation vector fields: $\nabla\times(\mathbf{Y}\times\mathbf{Z})$

$$\nabla\times(\mathbf{Y}\times\mathbf{Z})=\mathbf{Y}(\nabla\cdot\mathbf{Z})-\mathbf{Z}(\nabla\cdot\mathbf{Y})+(\mathbf{Z}\cdot\nabla)\mathbf{Y}-(\mathbf{Y}\cdot\nabla)\mathbf{Z} \quad (2)$$

Substituting ($Y = S$, $Z = A_g$), ($Y = S$, $Z = g$), and ($Y = S$, $Z = B_g$), Equation (2) gives NEW gravitation field equation:

$$\nabla\times(\mathbf{S}\times\mathbf{A}_g)=\mathbf{S}(\nabla\cdot\mathbf{A}_g)-(\mathbf{S}\cdot\nabla)\,\mathbf{A}_g. \quad (4.89)$$

$$\nabla\times(\mathbf{S}\times\mathbf{g})=\rho_g\,\mathbf{S}-(\mathbf{S}\cdot\nabla)\mathbf{g}. \quad (4.90)$$

$$\nabla\times(\mathbf{S}\times\mathbf{B}_g)=-(\mathbf{S}\cdot\nabla)\,\mathbf{B}_g. \quad (4.91)$$

Divergence of product of gravitation scalar potential φ_g and Spin S

$$\nabla\cdot(\varphi\mathbf{Y})=\varphi(\nabla\cdot\mathbf{Y})+\mathbf{Y}\cdot(\nabla\varphi) \quad (7)$$

Substituting Y = S into Equation (7), we obtain NEW gravitation field equations:

$$\nabla\cdot(\varphi_g\,\mathbf{S})=\mathbf{S}\cdot(\nabla\varphi_g) \quad (4.92)$$

Curl of product of scalar field φ and Spin S

$$\nabla\times(\varphi\mathbf{Y})=\varphi(\nabla\times\mathbf{Y})+(\nabla\varphi)\times\mathbf{Y}. \quad (8)$$

Substituting Y = S into Equation (8), we obtain NEW gravitation field equations:

$$\nabla\times(\varphi_g\,\mathbf{S})=(\nabla\varphi_g)\times\mathbf{S} \quad (4.93)$$

Gradient of scalar product of Spin and gravitation vector fields: $\nabla(\mathbf{S}\cdot\mathbf{A}_g)$, $\nabla(\mathbf{S}\cdot\mathbf{g})$, **And** $\nabla(\mathbf{S}\cdot\mathbf{B}_g)$

$$\nabla(\mathbf{Y}\cdot\mathbf{Z})=(\mathbf{Y}\cdot\nabla)\mathbf{Z}+(\mathbf{Z}\cdot\nabla)\mathbf{Y}+\mathbf{Y}\times(\nabla\times\mathbf{Z})+\mathbf{Z}\times(\nabla\times\mathbf{Y}). \quad (9)$$

Substituting ($Y = S$, $Z=A_g$), ($Y = S$, $Z=g$), and ($Y = S$, $Z=B_g$), into Equation (9), respectively, we obtain NEW gravitation field equation:

$$\nabla(\mathbf{S}\cdot\mathbf{A}_g)=(\mathbf{S}\cdot\nabla)\,\mathbf{A}_g+\mathbf{S}\times\mathbf{B}_g \quad (4.94)$$

$$\nabla(\mathbf{S}\cdot\mathbf{g})=(\mathbf{S}\cdot\nabla)\mathbf{g}+\mathbf{S}\times(\nabla\times\mathbf{g}). \quad (4.95)$$

$$\nabla(\mathbf{S}\cdot\mathbf{B}_g)=(\mathbf{S}\cdot\nabla)\,\mathbf{B}_g+\mathbf{S}\times(\nabla\times\mathbf{B}_g). \quad (4.96)$$

Cross Product: $(\mathbf{Y}\times\nabla)\cdot\mathbf{Z}$

$$(\mathbf{Y}\times\nabla)\cdot\mathbf{Z}=\mathbf{Y}\cdot(\nabla\times\mathbf{Z}). \quad (10)$$

Substituting ($\mathbf{Y}=\mathbf{S}$, $\mathbf{Z}=\mathbf{A}_g$), ($\mathbf{Y}=\mathbf{S},\mathbf{Z}=\mathbf{g}$), and ($\mathbf{Y}=\mathbf{S},\mathbf{Z}=\mathbf{B}_g$), respectively, Equation (10) gives new gravitation field equation,

$$(\mathbf{S}\times\nabla)\cdot\mathbf{A}_g=\mathbf{S}\cdot\mathbf{B}_g. \quad (4.97)$$

$$(\mathbf{S}\times\nabla)\cdot\mathbf{g}=-\mathbf{S}\cdot\partial(\mathbf{B}_g)/\partial\mathbf{t}. \quad (4.98)$$

$$(\mathbf{S}\times\nabla)\cdot\mathbf{B}_g=\mathbf{S}\cdot\mathbf{j}_g+\mathbf{S}\cdot\partial(\mathbf{g})/\partial\mathbf{t}. \quad (4.99)$$

Cross Product: $(\mathbf{Y}\times\nabla)\times\mathbf{Z}$

$$(\mathbf{Y}\times\nabla)\times\mathbf{Z}=\mathbf{Y}\times(\nabla\times\mathbf{Z})+(\mathbf{Y}\cdot\nabla)\mathbf{Z}-\mathbf{Y}(\nabla\cdot\mathbf{Z}) \quad (11)$$

Substituting ($\mathbf{Y}=\mathbf{S}$, $\mathbf{Z}=\mathbf{A}_g$), ($\mathbf{Y}=\mathbf{S},\mathbf{Z}=\mathbf{g}$), and ($\mathbf{Y}=\mathbf{S},\mathbf{Z}=\mathbf{B}_g$), respectively, Equation (11) gives new gravitation field equations:

$$(\mathbf{S}\times\nabla)\times\mathbf{A}_g=\mathbf{S}\times\mathbf{B}_g+(\mathbf{S}\cdot\nabla)\,\mathbf{A}_g-\mathbf{S}(\nabla\cdot\mathbf{A}_g). \quad (4.100)$$

$$(\mathbf{S}\times\nabla)\times\mathbf{g}=\mathbf{S}\times(\nabla\times\mathbf{g})+(\mathbf{S}\cdot\nabla)\mathbf{g}-\rho_g\,\mathbf{S}. \quad (4.101)$$

$$(\mathbf{S}\times\nabla)\times\mathbf{B}_g=\mathbf{S}\times(\nabla\times\mathbf{B}_g)+(\mathbf{S}\cdot\nabla)\,\mathbf{B}_g. \quad (4.102)$$

Summery

In this article, we show that the vector calculus identities are powerful tool in both rederiving the existing physical law, such as Maxwell equations, and discovering ssnew physical laws.

The mathematically derived new physical laws need to be tested experimentally.

References

1. Vector calculus identities, Wikipedia..