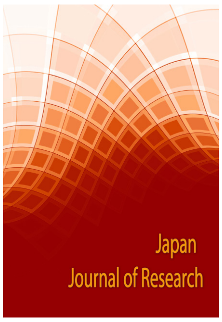
Japan Journal of Research

Opinion



Is ½ mv^2 the Kinetic Energy or One-Half of the Kinetic Energy?

Xiao En Wang[1,2], Deng Guo Wang[2], Yin Bo Wang[3]

[1]Weifang Engineering Vocational College (Weifang Educational College), Qingzhou City, Shandong Weifang, China. 262500.

[2]Zhaoqing KAIST battery material Co., Ltd. Gaoyao District of Zhaoqing City, Guangdong Province, China. 526100.

[3] 2023 Materials Major, College of Chemistry and Chemical Enginering, Qufu Normal University, Shandong, China 273165.

Correspondence

Xiao En Wang

Weifang Engineering Vocational College (Weifang Educational College), Qingzhou City, Shandong Weifang, China. 262500

ORCID Id: 0000-0001-8397-0640

Abstract

In the 1680s, Leibniz experimentally proved that vis viva is the product of the mass and square of velocity, mv^2. In 1847, Helmholtz studied free fall and found that the work produced or consumed was $(1/2)mv^2$ = mgh, and suggested calling the work as vis viva. By the end of the 19th century, people were still unable to derive (1/2)mv2 from the vis viva, mv^2, so they proposed to forget the old word 'vis viva' and create a new word 'kinetic energy', $(1/2)mv^2$, to replace the old word 'vis viva'. This article finds that $(1/2)mv^2$ is not a single valued function and cannot represent the kinetic energy. It can only represent one-half of the kinetic energy. And it was found that the kinetic energy in free fall motion is twice the potential energy, and the two cannot be converted into each other, and the correct expression of mechanical energy conservation was proposed.

- Received Date: 19 Mar 2026
- Accepted Date: 31 Mar 2026
- Publication Date: 07 Mar 2026

Keywords

kinetic energy; vis viva; potential energy; work; mechanical energy conservation

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The *vis viva* and Kinetic Energy in the History of Science

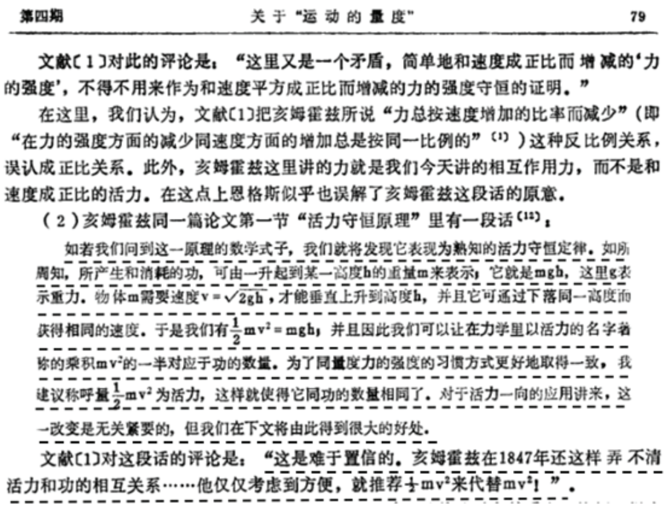
Today's students or scholars have never doubted the kinetic energy formula $(1/2)mv^2$. Last year, we borrowed a high school physics textbook [1] from Teacher Huang at Changtian Middle School in Zhaoqing. By it, we saw a different formula for kinetic energy. As shown in Figure-1, the book introduces the "historical debates on motion measurement issues" on page 4 in the form of a titled "Loose-leaf of Data", mainly focusing on momentum and kinetic energy. However, it also introduced the discourse of German scientist G. Leibniz (1646-1716) on kinetic energy (at that time, it was called *vis viva*). The underlined part in the figure-1 is now excerpted and translated as follows:

> "He believed that the effects of letting a 1-pound (1-pound=0.4536 kilograms) object fall freely by 4 feet (1-foot=0.3048 meters) and letting a 4-pound object fall freely by 1 foot were the same in both cases, as they caused the same deformation when they fell to the ground. However, the momentum mv obtained from these two free fall movements was not equal, but the product of mass and velocity square mv^2 was equal. Therefore, mv^2 should be used as a measure of motion and referred to as *vis viva*. Later, according to Coriolis's suggestion, the *vis viva* of a moving object, mv^2, was replaced by the ½ mv^2, which later became known as the kinetic energy of a moving object."

The textbook also introduces the evaluations of French scientist J. d'Alembert (1717-1783) and German philosopher F. Engels (1820-1895) on that historic debate. In order to understand Engels' detailed evaluation of that debate, we found an article titled "On the Measurement of Motion"[2] published by Professor Hong Guan from the Physics Department of Sun Yat sen University in 1983. As shown in Figure-2, Hong Guan's article quotes a passage from German scientist Hermann von Helmholtz (1821-1894) and Engels' evaluation of this passage. The underlined part in the Figure-2 is now excerpted and translated as follows:

> "If we ask for the mathematical formula of this principle, we will find that it manifests as the well-known law of conservation of *vis viva*. As is well known, the work produced and consumed can be represented by the weight m that rises to a certain height h, which is mgh, where g represents gravity. Object m needs a velocity of $v = \sqrt{2gh}$ to rise vertically to height h, and it can achieve the same velocity by falling to the same height. So we have $(1/2)mv^2 = mgh$, and therefore we can make one-half of the product mv^2 known in mechanics as *vis viva* correspond to the amount of work. In order to achieve better consistency in the habitual way of measuring the strength of the same force, I suggest calling $(1/2)mv^2$ as *vis viva*, so that this makes it equal to the value of work. For the application of *vis*

Citation: Wang XE, Wang DG, Wang YB, Is ½ mv² the Kinetic Energy or One-Half of the Kinetic Energy?. Japan J Res. 2026;7(3):179



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文献〔1〕对此的评论是：“这里又是一个矛盾，简单地和速度成正比而增减的‘力的强度’，不得不用来作为和速度平方成正比而增减的力的强度守恒的证明。”

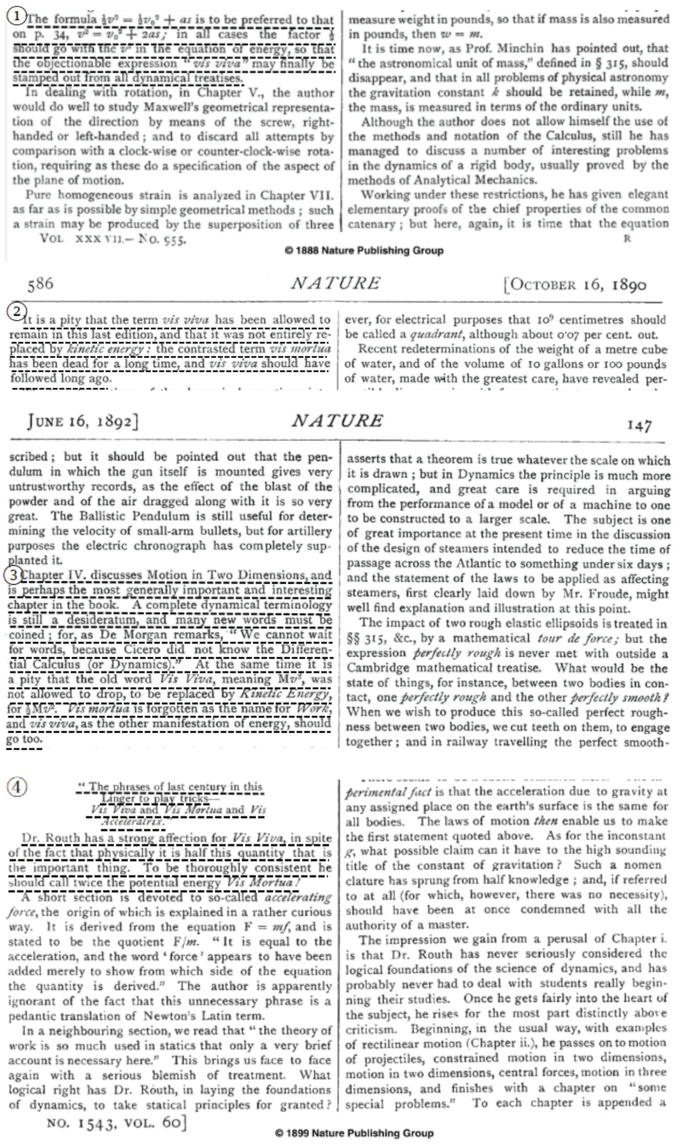
在这里，我们认为，文献〔1〕把亥姆霍兹所说“力总按速度增加的比率而减少”（即“在力的强度方面的减少同速度方面的增加总是按同一比例的”〔1〕）这种反比例关系，误认成正比关系。此外，亥姆霍兹这里讲的力就是我们今天讲的相互作用力，而不是和速度成正比的活力。在这点上恩格斯似乎也误解了亥姆霍兹这段话的原意。

（2）亥姆霍兹同一篇论文第一节“活力守恒原理”里有一段话〔12〕：

如若我们问到这一原理的数学式子，我们就将发现它表现为熟知的活力守恒定律。如所周知，所产生和消耗的功，可由一升起到某一高度h的重量m来表示；它就是mgh，这里g表示重力。物体m需要速度 $v=\sqrt{2gh}$，才能垂直上升到高度h，并且它可通过下落同一高度而获得相同的速度。于是我们有 $\frac{1}{2}mv^2=mgh$；并且因此我们可以让在力学里以活力的名字着称的乘积 mv^2 的一半对应于功的数量。为了同量度力的强度的习惯方式更好地取得一致，我建议称呼量 $\frac{1}{2}mv^2$ 为活力，这样就使得它同功的数量相同了。对于活力一向的应用讲来，这一改变是无关紧要的，但我们在下文将由此得到很大的好处。

文献〔1〕对这段话的评论是：“这是难于置信的。亥姆霍兹在1847年还这样弄不清活力和功的相互关系……他仅仅考虑到方便，就推荐 $\frac{1}{2}mv^2$ 来代替 mv^2！”。

Figure 1. *The relevant citations in Hong Guan 's article.*



①The formula $\frac{1}{2}v^2 = \frac{1}{2}v_0^2 + as$ is to be preferred to that on p. 34, $v^2 = v_0^2 + 2as$; in all cases the factor $\frac{1}{2}$ should go with the v^2 in the equation of energy, so that the objectionable expression "*vis viva*" may finally be stamped out from all dynamical treatises.

In dealing with rotation, in Chapter V., the author would do well to study Maxwell's geometrical representation of the direction by means of the screw, right-handed or left-handed; and to discard all attempts by comparison with a clock-wise or counter-clock-wise rotation, requiring as these do a specification of the aspect of the plane of motion.

Pure homogeneous strain is analyzed in Chapter VII. as far as is possible by simple geometrical methods; such a strain may be produced by the superposition of three

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measure weight in pounds, so that if mass is also measured in pounds, then $w = m$.

It is time now, as Prof. Minchin has pointed out, that "the astronomical unit of mass," defined in § 315, should disappear, and that in all problems of physical astronomy the gravitation constant k should be retained, while m, the mass, is measured in terms of the ordinary units.

Although the author does not allow himself the use of the methods and notation of the Calculus, still he has managed to discuss a number of interesting problems in the dynamics of a rigid body, usually proved by the methods of Analytical Mechanics.

Working under these restrictions, he has given elegant elementary proofs of the chief properties of the common catenary; but here, again, it is time that the equation

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②It is a pity that the term *vis viva* has been allowed to remain in this last edition, and that it was not entirely replaced by *kinetic energy*: the contrasted term *vis mortua* has been dead for a long time, and *vis viva* should have followed long ago.

ever, for electrical purposes that 10^9 centimetres should be called a *quadrant*, although about 0·07 per cent. out.

Recent redeterminations of the weight of a metre cube of water, and of the volume of 10 gallons or 100 pounds of water, made with the greatest care, have revealed per-

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scribed; but it should be pointed out that the pendulum in which the gun itself is mounted gives very untrustworthy records, as the effect of the blast of the powder and of the air dragged along with it is so very great. The Ballistic Pendulum is still useful for determining the velocity of small-arm bullets, but for artillery purposes the electric chronograph has completely supplanted it.

③Chapter IV. discusses Motion in Two Dimensions, and is perhaps the most generally important and interesting chapter in the book. A complete dynamical terminology is still a desideratum, and many new words must be coined; for, as De Morgan remarks, "We cannot wait for words, because Cicero did not know the Differential Calculus (or Dynamics)." At the same time it is a pity that the old word *Vis Viva*, meaning Mv^2, was not allowed to drop, to be replaced by *Kinetic Energy*, for $\frac{1}{2}Mv^2$. *Vis mortua* is forgotten as the name for *Work*, and *vis viva*, as the other manifestation of energy, should go too.

asserts that a theorem is true whatever the scale on which it is drawn; but in Dynamics the principle is much more complicated, and great care is required in arguing from the performance of a model or of a machine to one to be constructed to a larger scale. The subject is one of great importance at the present time in the discussion of the design of steamers intended to reduce the time of passage across the Atlantic to something under six days; and the statement of the laws to be applied as affecting steamers, first clearly laid down by Mr. Froude, might well find explanation and illustration at this point.

The impact of two rough elastic ellipsoids is treated in §§ 315, &c., by a mathematical *tour de force*; but the expression *perfectly rough* is never met with outside a Cambridge mathematical treatise. What would be the state of things, for instance, between two bodies in contact, one *perfectly rough* and the other *perfectly smooth*? When we wish to produce this so-called perfect roughness between two bodies, we cut teeth on them, to engage together; and in railway travelling the perfect smooth-

④"The phrases of last century in this
Linger to play tricks—
Vis Viva and *Vis Mortua* and *Vis Acceleratrix*."

Dr. Routh has a strong affection for *Vis Viva*, in spite of the fact that physically it is half this quantity that is the important thing. To be thoroughly consistent he should call twice the potential energy *Vis Mortua*!

A short section is devoted to so-called *accelerating force*, the origin of which is explained in a rather curious way. It is derived from the equation $F = mf$, and is stated to be the quotient F/m. "It is equal to the acceleration, and the word 'force' appears to have been added merely to show from which side of the equation the quantity is derived." The author is apparently ignorant of the fact that this unnecessary phrase is a pedantic translation of Newton's Latin term.

In a neighbouring section, we read that "the theory of work is so much used in statics that only a very brief account is necessary here." This brings us face to face again with a serious blemish of treatment. What logical right has Dr. Routh, in laying the foundations of dynamics, to take statical principles for granted?

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perimental fact is that the acceleration due to gravity at any assigned place on the earth's surface is the same for all bodies. The laws of motion *then* enable us to make the first statement quoted above. As for the inconstant g, what possible claim can it have to the high sounding title of the constant of gravitation? Such a nomenclature has sprung from half knowledge; and, if referred to at all (for which, however, there was no necessity), should have been at once condemned with all the authority of a master.

The impression we gain from a perusal of Chapter i. is that Dr. Routh has never seriously considered the logical foundations of the science of dynamics, and has probably never had to deal with students really beginning their studies. Once he gets fairly into the heart of the subject, he rises for the most part distinctly above criticism. Beginning, in the usual way, with examples of rectilinear motion (Chapter ii.), he passes on to motion of projectiles, constrained motion in two dimensions, motion in two dimensions, central forces, motion in three dimensions, and finishes with a chapter on "some special problems." To each chapter is appended a

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Figure 2. *①, ②, ③ and ④ are respectively excerpted from four articles in Nature*

viva, this change is insignificant, but we will benefit greatly in the following text. "[2]

In 1880 or 1881, Engels evaluated Helmholtz's statement: "It's unbelievable that Helmholtz couldn't figure out the relationship between *vis viva* and work in 1847. He recommended $(1/2)mv^2$ instead of mv^2 just for convenience! "[2].

To verify whether Engels' evaluation represented the mainstream view in the academic community at that time, we searched for a total of 131 articles in Nature from its founding in 1845 to 1900 using the keyword "*vis viva*". As shown in Figure 2, the underlined parts of the four of the 131 articles that have important discussions on the significance of *vis viva* are excerpted as follows:

①. The formula $½v^2 = ½v_0^2 + as$ is to be preferred to that on p. 34, $v^2 = v_0^2 + 2as$; in all cases the factor ½ should go with the v^2 in the equation of energy, so that the objectionable expression " *vis viva*" may finally be stamped out from all dynamical treatises.[3]（*Nature* 1888）

②. It is a pity that the term *vis viva* has been allowed to remain in this last edition, and that it was not entirely replaced by kinetic energy: the contrasted term *vis mortua* has been dead for a long time, and *vis viva* should have followed long ago. [4]（*Nature* 1890）

③. Chapter IV. discusses Motion in Two Dimensions, and is perhaps the most generally important and interesting chapter in the book. A complete dynamical terminology is still a desideratum, and many new words must be coined; for, as De Morgan remarks, "We cannot wait for words, because Cicero did not know the Differential Calculus (or Dynamics)." At the same time it is a pity that the old word *Vis Viva*, meaning Mv^2 was not allowed to drop, to be replaced by *Kinetic Energy*, for $½Mv^2$. *Vis mortua* is forgotten as the name for Work, and *vis viva*, as the other manifestation of energy, should go too.[5] (*Nature* 1892)

④. Dr. Routh has a strong affection for *Vis Viva*, in spite of the fact that physically it is half this quantity that is the important thing. To be thoroughly consistent he should call twice the potential energy *Vis Mortua* ! [6] (*Nature* 1899)

By the above historical documents, it can be seen that the academic community did not complete the theoretical transition and derivation from *vis viva, mv^2*, to the new term kinetic energy $(1/2)mv^2$ until the end of the 19th century. That is to say, the mainstream academic community is clear about the difference between work $(1/2)mv^2$ and *vis viva, mv^2*. The *vis viva* cannot be reduced to the work, nor can it be replaced by work. The above historical data proves that Engels' criticism of Helmholtz indeed represented the mainstream academic viewpoint at that time. It can also be seen that J.G. MacGregor et al., due to their inability to theoretically derive and transition from work $(1/2)mv^2$ to *vis viva, mv^2*, could only forget the old word *vis viva, mv^2* and create a new word kinetic energy $(1/2)mv^2$. The kinetic energy of creating new words is equal to work, but not equal to *vis viva*. However, today, when people look back at history, they have already regarded *vis viva* and later kinetic energy as the same physical concept. Although they have different mathematical expressions, they have been confused since Helmholtz.

Forgetting old things and instilling new ones is certainly standard brainwashing today. Brainwashing is a method of forcibly changing people's minds within some pyramid schemes that have been banned by the Chinese government. It can be said that in the late 19th century, a generation of scientists changed an important physical concept and mathematical formula using non scientific methods. This unresolved legacy issue in history can be reanalyzed and discussed today. In terms of concept and definition, the *vis viva* and the kinetic energy both refer to the energy possessed by an object due to its motion. They should only be the relationship between old words and new words, and should have the same mathematical expression, rather than two expressions, $m\upsilon^2$ and $(1/2)m\upsilon^2$. As shown in equation (1), $(1/2)\ m\upsilon^2$ is originally a mathematical expression for calculating work, and all terms in equation (1) represent work.

In history, Helmholtz renamed work $(1/2)m\upsilon^2$ as *vis viva* and stated that this change is irrelevant. His change is the root of all the confusion in dynamics teaching to this day.

2. **It can be proven that $(1/2)m\upsilon^2$ is work rather than kinetic energy**

$$W=F\cdot L=mg\cdot h=mg\cdot(1/2)\upsilon t=m(\upsilon/t)\cdot(1/2)\upsilon t=(1/2)m\upsilon^2 \qquad (1)$$

They are all work!!

Let's first analyze the nature of $(1/2)m\upsilon^2$ during a non-uniform acceleration process. Because m can usually be regarded as a constant, we only analyze the changes in specific work W/m and specific kinetic energy T/m. The kinetic energy is a state function and an instantaneous quantity, but the work is not a state function and is a process quantity. For comparison, we take the work done per unit time.

Figure 4A shows a velocity curve that transitions from uniform acceleration to non-uniform acceleration and then gradually transitions to uniform motion. The acceleration process is uniform during from 0 to 1 second, gradually decreasing acceleration from 1 to 3 seconds, and uniform motion after 3 seconds. Figure 4B shows the curve corresponding to the work done per unit time in the process represented by Figure 4A. Near point *c* (2s), although the speed is still increasing, the work done per unit time has begun to decrease. Arriving at point d and entering the stage of uniform motion, the work done per unit time has almost decreased to zero.

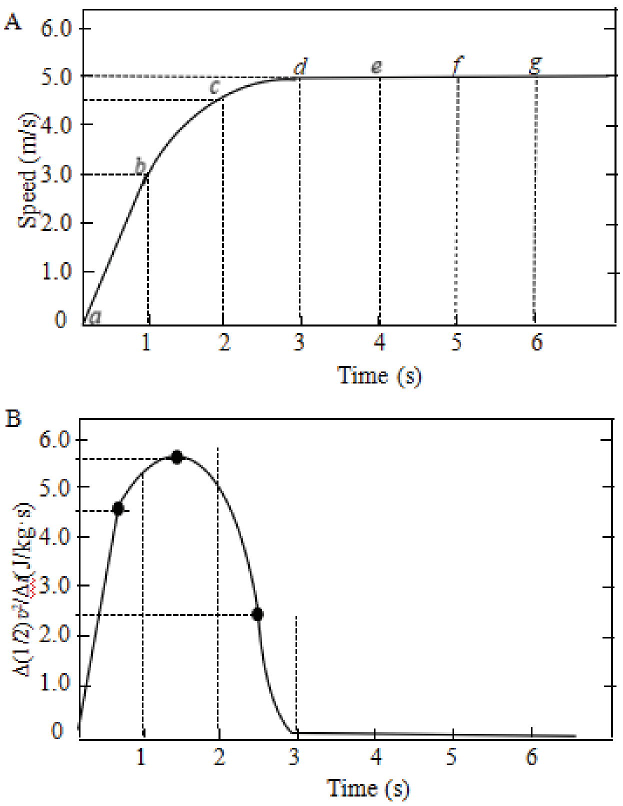

Fig. 4. A. The speed curve representing a variable speed process; B. $\Delta(1/2)\upsilon^2/\Delta t$ correspond to the curve of work done per unit time.

The data analysis of the process shown in Figure 4 is listed in Table 1, and the work done per unit time corresponds to $\Delta(1/2)\upsilon^2/\Delta t$. For a gradually decreasing acceleration process, the work done per unit time decreases. When entering the stage of uniform motion, no more work is done per unit time, that is, $\Delta(1/2)\upsilon^2/\Delta t \rightarrow 0$. However, at any moment of uniform motion, its kinetic energy cannot be zero. Therefore, only the product of υ^2 and mass m can represent the kinetic energy. The product of $(1/2)\upsilon^2$ and mass m at any stage cannot represent the kinetic energy, but only the work done during that stage.

Obviously, work and *vis viva* (kinetic energy) are fundamentally different, and Helmholtz did not see the fundamental difference between work and *vis viva*. He gave the example of free fall motion at that time, and now we will analyze it in terms of free fall motion. We start from the definition of work to calculate the work W done during the falling process:

The *mgh* is not suitable as potential energy

The *mgh* in equation (1) represents work, but it looks a bit

Table 1. *Exploring the Essence of υ^2 and $(1/2)\upsilon^2$*

Point	a	b	c	d	e	f	g
Time(s)	0	1	2	3	4	5	6
Speed (m/s)	0	3.0	4.5	5.0	5.0	5.0	5.0
υ^2	0	9	20.25	25.0	25.0	25.0	25.0
$\Delta(1/2)\upsilon^2/\Delta t$	0	4.5	5.625	2.375	0	0	0

like potential energy, and even people today directly consider it as potential energy. Potential energy definitely refers to the potential energy of gravity, which can be approximated as the gravitational potential energy of the Earth. If h represents high, it is not potential energy. There are several reasons for this:

1. Height is a habitual concept that has been formed in people's daily lives, usually with the ground as the zero point and pointing upwards, opposite to the direction of gravity. According to the definition of work, work is the product of force and the displacement distance of an object along the direction of force. The use of habitual concepts (height) to calculate the work done by free fall has resulted in confusing physical concepts.
2. If *mgh* represents potential energy and h represents height, then the larger h, the greater the potential energy. This contradicts the fact that the higher the position, the smaller the gravitational potential energy of the Earth, and the potential energy is zero at infinity. The higher (or farther) the position, the greater the potential energy, which is not scientific. This sentence can only be said when the potential energy takes a negative value, but high h also needs to be represented by a negative value. Physics teachers often use the concept of negative potential energy to cover up their mistakes.
3. In current physics, the height is calculated as $h=(1/2)\ gt^2$, where h increases with the square of time t, indicating that the object gradually rises throughout the entire process of time, contrary to the assumed free fall process. If h is considered high, *mgh* is not the work done by free fall

motion, but rather the work required for an ascending process. Only by considering h as the falling displacement of an object can mgh represent the potential energy of the object.

4. The current textbook regards mgh as potential energy and believes that potential energy and kinetic energy are converted into each other during the falling process, which is incorrect. According to this theory, the maximum potential energy of an object at the beginning of its decline corresponds to the maximum velocity at which the object is transformed; When approaching the ground, the potential energy is about to become zero, corresponding to the minimum speed. The fact is exactly the opposite.
5. How can an object know whether it is going to land on a high mountain or a deep ditch before it falls? How can it prepare a definite potential energy that is exactly converted into kinetic energy when it reaches the ground? There are two apples of the same size at the same height on a tree, they should have the same potential energy! When they land, one lands on the ground and the other faces a well. Has all the potential energy it has prepared been converted into kinetic energy? Should it suspend at the wellhead and not fall down?

It can be seen that the use of high h in the potential energy mgh is incorrect and misleading. It misleads the idea of the mutual conversion between potential energy and kinetic energy. Of course, whether considering h as high or displacement, the change in work (W) and potential energy (ΔV) is numerically equal.

(1/2)mv^2 in equation (1) is not suitable for expression of kinetic energy

1. One of the authors of this article, Xiaoen Wang had already raised the viewpoint that $(1/2)mv^2$ is not suitable for expressing kinetic energy in an article [**7**] published in 2024. Now a more in-depth brief explanation is as follows.
2. Undoubtedly, the $(1/2)mv^2$ is completely correct for calculating the work of free fall process. Work is a process quantity, not a state function. And kinetic energy (or *vis viva*) is a state function, an instantaneous quantity. By equation (1), it can be seen that the $(1/2)mv^2$ is a multivalued function composed of two states (two velocities) of an object. It corresponds to a maximum instantaneous velocity, v, and another average velocity, $v/2$. A multivalued function cannot be a state function, therefore, the $(1/2)mv^2$ is not a state function, and therefore cannot be used as a mathematical expression for kinetic energy (or *vis viva*).
3. The $(1/2)mv^2$ is not kinetic energy, but can be considered as one-half of the kinetic energy, mv^2. The mv^2 is a single valued function, which also corresponds to a single valued instantaneous motion state with a velocity of v. In this way, equation (1) represents that the work of free fall motion is equal to one-half of the kinetic energy (or *vis viva*), mv^2.
4. Comparison between $(1/2)mv^2$ and $(1/2)\,gt^2$: If the former is viewed as a global function, it is a binary-valued function. The v represents two velocity states: the highest velocity, v, and the average velocity, $v/2$. For the latter, when g is treated as a constant, t is single valued. Therefore, in equation $(1/2)\,gt^2=(1/2)\,v{\cdot}t = h$, all three terms are also single valued. Although $v/2$ represents the average velocity, the v has a clear and unique physical meaning, referring to the instantaneous velocity of the process or the highest velocity before landing. For the $v/2$, you can tell at a glance that it's one-half of the speed, and will not cause misunderstandings.

The current definition of kinetic energy is the energy that an object possesses due to its motion. Kinetic energy is a function of the state of a moving object. A moving object has a corresponding instantaneous velocity at every moment, and it must also have an instantaneous kinetic energy. From the fifth term $m\,(v/t)\cdot(1/2)\,vt$ in equation (1), it can be seen that the work $(1/2)mv^2$ is the product of the mass m， the maximum velocity v and the average velocity $v/2$. Work is not a state function, its magnitude is determined by two instantaneous velocities (maximum velocity and average velocity) in two motion states, and $(1/2)mv^2$ does not describe the physical quantity of a moving object in the same motion state. Therefore, theoretically speaking, the $(1/2)mv^2$ is definitely not kinetic energy, because it cannot correspond to a definite instantaneous velocity, that is, it cannot correspond to the kinetic energy of a definite motion state. According to the definition of kinetic energy, the $(1/2)mv^2$ can only be one-half of the kinetic energy, mv^2, and only the mv^2 can correspond to the kinetic energy at a certain motion state (determined instantaneous velocity, v).

The mathematical expression of Work-kinetic Energy Theorem, $W = (1/2)\,mv^2$, in current textbooks is still correct, but the right-hand side of the equation should be interpreted as one-half of the kinetic energy. That is, the work done during a free-falling body is equal to one-half of the kinetic energy at speed v, not equal to the kinetic energy.

Revised new mechanical energy conservation relationship

The late 19th century was a critical era for physics to move from conceptual confusion to finalization. The transition from most old words to new words is reasonable. For example, the '*work*' emerged as a new term in the 19th century, corresponding to the old word '*vis mortua*'. In Greenhill's article in 1892 '*vis mortua*' was used to represent 'work'. In C.G.K.'s article in 1899, '*vis mortua'* was equated with potential energy. And he also pointed out that *vis viva* is twice the potential energy. Regardless of whether the old term '*vis mortua*' in history represents work or potential energy, because they are equal in mathematical formulas or numerical values, they are replaced by the new terms 'work' and 'potential energy', and theoretically there is no problem.

The $W = mgh = (1/2)\,mv^2$ in equation (1) is the root of all confusion in the teaching of conservation of mechanical energy. The three terms connected by two equal signs represent work, the mgh is not the gravitational potential energy of the Earth, and the $(1/2)\,mv^2$ is not the kinetic energy of free fall. For potential energy, using high to represent the trajectory of a falling object is not appropriate, as high is exactly opposite to the starting point, direction of motion, and endpoint of the falling trajectory. In order to avoid confusion and misunderstanding, we suggest using the falling displacement l of free fall instead of high h. The zero point of l is the starting point of the fall, and the direction of l is downward, consistent with the direction of gravity.

Based on the above analysis, theoretically the $(1/2)\,mv^2$ should be considered as one-half of the kinetic energy, mv^2, and the kinetic energy, mv^2, should be equal to twice the work or twice the potential energy. So, by multiplying both sides of equation (1) by 2, we obtain

$$2W = 2mgl = mv^2 \text{ or } 2W = 2V = T \quad (V \geqslant 0) \tag{2}$$

where V and T represent potential energy and kinetic energy, respectively, and $V= mgl$ and $T=mv^2$. According to equation (1), $W = V$, or $2W=W+V$, so the second equation in equation (2) can be written as

$$W + V = T \tag{3}$$

or $T + (-V) = W \quad (V \geq 0)$ (4)

In order to follow traditional customs, we still refer to the left side of equation (4) as mechanical energy, which is the sum of the kinetic energy and the potential energy (potential energy takes a negative value). So we have

$$\begin{cases} T + (-V) = W, & (W \neq 0;\ V \geq 0) \quad (5\text{-}1) \\ T + (-V) = 0, \text{ or } T = V & (W = 0;\ V \geq 0) \quad (5\text{-}2) \end{cases}$$

Based on a new derivation, the equations (5-1) and (5-2) are mathematical expressions for a new conservation of mechanical energy. Equation (5-1) indicates that when a system exchanges energy in the form of work with the external environment, the mechanical energy of the system equals the work; Equation (5-2) indicates that if there is no energy exchange in the form of work between the system and the outside world, the total mechanical energy of the system is zero, or the kinetic energy is equal to the potential energy (which takes a positive value).

Re evaluation of the history of science

Looking back at the history of the 19th century's science , the old term '*vis viva*' and the new term 'kinetic energy' should not have much difference in physical concepts, as they both describe the state functions of energy of moving objects. But two completely different mathematical expressions, mv^2 and (1/2) mv^2, emerged, and only one of the two mathematical expressions must be reasonable and correct. Helmholtz not only suggested calling (1/2)mv^2 as *vis viva* and using it to replace the mv^2, but also claimed that this change is irrelevant. Is it really irrelevant? Is it irrelevant to change a quantity to one-half of it? J. G. MacGregor *et al*. created new words to make people forget old words and change a quantity to one-half of it. Is this kind of physics still considered scientific?

Whether the kinetic energy is equal to the potential energy or work, or is twice the potential energy or twice work, is a significant event in theoretical physics. Of course, in the Helmholtz era, considering the (1/2) mv^2 in the work-kinetic energy theorem as the kinetic energy or as one-half of the kinetic energy, the result is the same when calculating the value of work. Due to the abstraction of the kinetic energy, Helmholtz also bullied the kinetic energy too much, and he forcibly cut it to its one-half.

The work-kinetic energy theorem in current textbooks is included in equation (1)

$$W = (1/2)mv^2 \qquad (1\text{-}1)$$

Although we have pointed out above that the (1/2) mv^2 is not a single valued function and cannot be used as an expression for the kinetic energy, it can only be used as a formula for calculating work. The same work-kinetic energy theorem, if we understand the (1/2) mv^2 in equation (1) as one-half of the kinetic energy, mv^2, theoretically there is no problem at all. Helmholtz referred to the (1/2) mv^2 in the formula (1/2) $mv^2 = mgh$ as kinetic energy, theoretically leading to the first mistake of "kinetic energy equals potential energy in free fall". Also, due to the incorrect use of the conventional physical concept of high h for the displacement (distance) of free fall, the potential energy of the starting point and landing point was reversed, which is the second mistake. These two errors together led to the erroneous conclusion that potential energy and kinetic energy are converted into each other during free fall, and have been crowned with the golden signboard of energy conservation. For over a hundred years, people have forgotten that there was once a correct physics theory, and no one has ever doubted the current erroneous textbooks.

Fields affected by current incorrect kinetic energy formulas

Entering the 20th century, with the rapid development of science and technology, the issue that is it the kinetic energy or one-half of the kinetic energy will definitely have a noticeable and significant impact. The Kepler system is the best natural laboratory for verifying theories of mechanics and dynamics. Furthermore, errors are bound to occur in the conversion of other forms of energy into the kinetic energy, especially in the field of calculating velocity by the kinetic energy. After careful research, we have finally discovered the impact of this error from the following aspects.

1) Doppler system

The equations (5-1) and (5-2) that represent the conservation of mechanical energy we obtained are completely consistent with the energy relationship of the Kepler system [8]. Energy is a more abstract physical quantity than force, so our consideration starts with simpler and more intuitive concepts such as universal gravitation and centrifugal force. In the orbital motion of planets around the Sun in the solar system, each planet experiences equal gravitational and centrifugal forces. namely

$$GMm/R^2 = mv^2/R \qquad (6)$$

where G, M, m, R, and v represent the gravitational constant, solar mass, planetary mass, orbital radius, and planetary revolution speed, respectively. Multiplying both sides of equation (6) by R yields

$$GMm/R = mv^2 \text{ or } V = T \quad (W=0) \qquad (7)$$

In equation (7), the left side (GMm/R) represents the potential energy of the planet in the gravitational field of the sun; On the right (mv^2) is the kinetic energy of the planet. The kinetic energy and the potential energy here are equal, which fully conforms to the equation (5-2) derived in this article. However, it's not the same thing compared to the current theory that the kinetic energy and potential energy in free fall are equal and can be converted into each other. Because free fall is a process of doing work, but the Kepler system did not exchange energy with the outside world in the form of work. So the kinetic energy of planetary revolution is equal to its potential energy. The kinetic energy equals the potential energy, what a simple and beautiful natural law it is! The conservation of mechanical energy in current textbooks only states that the sum of kinetic energy and potential energy is equal to a constant.

Kepler's third law can be derived from equation (7). We use the orbital radius R and period f to represent the orbital velocity v, $v = 2\pi R/f$. By equation (7), we obtain Kepler's third law

$$GM = Rv^2 = R\cdot(2\pi R/f)^2 = 4\pi^2R^3/f^2, \text{ or } R^3/f^2 = GM/(4\pi^2) = \text{constant} \qquad (8)$$

Let's take the Earth as an example to illustrate the changes in the kinetic energy and the potential energy when moving in an elliptical orbit. The elliptical orbital motion follows the conservation of angular momentum, i.e. mvR = constant. When the Earth moves from the vernal equinox to the aphelion, the orbital radius increases and the velocity decreases, both the Earth's potential energy and kinetic energy will decrease; When the Earth moves from the autumnal equinox to perihelion, the orbital radius decreases and the velocity increases, both the Earth's potential energy and kinetic energy will increase. In short, when a planet orbits the sun or a satellite orbits the earth in a circular motion, kinetic energy equals potential energy; When a planet or satellite undergoes elliptical motion, both kinetic and potential energy decrease or increase simultaneously, and follow the conservation of orbital angular momentum. There is no natural existence of the (1/2) mv^2 in the entire Kepler system. The Kepler system validated the new law of conservation of mechanical energy represented by (5-1) and (5-2). Based on the Kepler system, it has been proven that the theory of mutual transformation between kinetic energy and potential energy is incorrect.

2) Second cosmic velocity

Flying, landing on the moon, and capturing stars have always been human dreams. In the first half of the 20th century, people discovered that speed is the key to realizing humanity's dream of flying into space [9]. Among them, the first cosmic velocity υ_1=7.9 kilometers per second and the second cosmic velocity υ_2=11.2 kilometers per second are two important characteristic velocities, $\upsilon_2 = \sqrt{2}\ \upsilon_1$. The first cosmic velocity is calculated based on force balance. Both sides of equation (6) multiplied by *R/m,* we get $GM/R = \upsilon^2$, where R represents the average radius of the Earth and M represents the mass of the Earth. So the lowest velocity for orbiting the Earth, υ_1, is

$$v_1 = \sqrt{\frac{GM}{R}} = \sqrt{\frac{6.67\times10^{-11}\times5.965\times10^{24}}{6.371\times10^{6}}} = 7.90\times10^{3}\,\mathrm{m/s}\ \text{or}\ 7.90\ \mathrm{km/s}. \qquad (9)$$

The calculation formula for the first cosmic velocity not only does not involve the current kinetic energy formula, (1/2) $m\upsilon^2$, but also includes the mathematical relationship shown in equation (7) that the potential energy equals the kinetic energy (*vis viva,* $m\upsilon^2$). As long as the kinetic energy formula ($m\upsilon^2$) is correct, the theoretical first cosmic velocity must be consistent with the measured velocity.

The second cosmic velocity is calculated according to the current kinetic energy formula (1/2)$m\upsilon^2$. In order to clarify people's vague understanding of the meanings of these two velocities, Mr. Dayao Li from the Beijing Institute of Space Mechanical and Electrical Research pointed out in an article [10] published in the journal Spacecraft Recovery & Remote Sensing in 2003 that these two velocities "are two concepts introduced by people to estimate the minimum energy required to overcome the gravitational constraints of the Earth (represented by the ratio of the mechanical energy of the object relative to the center of mass of the Earth to the mass of the object - specific mechanical energy)." Mr. Li wrote in the following fourth part Second Cosmic Velocity and Escape Velocity that the second cosmic velocity "is calculated based on the initial kinetic energy of a spacecraft that can completely offset the work done by the Earth's gravity on it during its flight to infinity from the center of the Earth".

Due to the current theory that (1/2) $m\upsilon^2$ is the kinetic energy, if the flight speed is designed according to energy conservation, consuming (1/2)$m\upsilon^2$ of other energy should be able to convert it into an equal amount of kinetic energy, thereby achieving velocity υ. However, in reality, it may only reach $\upsilon/\sqrt{2}$. That is to say, using the energy conservation scheme, even considering the thermal efficiency of rocket engines, the designed speed cannot actually reach υ. This may be the real reason behind the proposal of the second cosmic velocity. Of course, there may be multiple reasons why the actual speed cannot reach the design speed, such as atmospheric resistance.

In the mid-20th century, when humans first began exploring space flight, the heat released from fuel, according to the law of conservation of energy, from thermal energy to kinetic energy, and then to velocity, may be the preferred design solution for designers. Therefore, the second cosmic velocity may be a remedial velocity when using energy design schemes.

3) Rocket for launching artificial satellites

Human exploration of space began during the Cold War era, the earliest rockets used by the former Soviet Union and the United States to launch satellites were based on intercontinental missiles. The detailed information of rockets related to satellite launches is also a military secret, and specific parameters are still difficult to find. Now, using AI, the publicly available data of the first artificial Earth satellite from the former Soviet Union, the United States, and China can be found online as follows.

On October 4, 1957, the former Soviet Union successfully launched the world's first artificial satellite, Sputnik I. The satellite weighs 83.6 kilograms, with a perigee of 228 kilometers and an apogee of 947 kilometers. It has an operating period of 96.17 minutes and has been in orbit for 92 days. It re entered the atmosphere and burned up on January 4, 1958.

On January 31, 1958, the United States launched its first satellite, Explorer 1. The satellite weighs 8.2 kilograms, with an orbital perigee of 358 kilometers and an apogee of 2550 kilometers, and an orbital period of 114.8 minutes. After operating in orbit for over 12 years, it re entered the atmosphere and burned up on March 31, 1970.

On April 24, 1970, China launched its first satellite, Dongfanghong-1, with a weight of 173 kilograms, a perigee of 441 kilometers, an apogee of 2368 kilometers, and an orbital period of 114 minutes. As of February 2009, nearly 40 years later, it is still in orbit with a perigee of 430 kilometers and an aphelion of 2075 kilometers. On the 55th Anniversary of Flying in the Sky on April 24, 2025, Houren Pan, the deputy head of the overall design team of Dongfanghong-1, said that Dongfanghong-1 is expected to "live" until the age of 100.

The lifespan of the three artificial satellites mentioned above is 92 days, 12 years, and approximately 100 years, respectively. Why was the lifespan of the first artificial satellite in the former Soviet Union so short? It is likely that the actual speed has not reached the design speed and differs significantly from the actual speed. If designers refer to the energy scheme of releasing energy from fuel → kinetic energy → velocity, the design velocity will naturally be greatly reduced.

Table 2 lists the reference data for each sub stage of the Long March 1 rocket that launched China's Dongfanghong-1 satellite. The main dynamic parameters used by the Long March 1 rocket are sea level thrust and sea level specific impulse. That is to say, the momentum impulse scheme was chosen instead of the energy conservation scheme containing kinetic energy terms. According to the impulse theorem in classical mechanics, the impulse I of the external force acting on an object is equal to the increment Δp of its momentum. Momentum is a state quantity, while impulse is a process quantity. Usually, the fuel combustion gas nozzle area of a rocket is fixed, and the flow rate of the ejected gas is also constant, so the thrust of the rocket is usually constant during operation. The impulse of constant thrust is the product of thrust and its duration of action. There are the following relationships between them.

$$\mathrm{I} = \int F dt = \Delta p = \int \frac{dp}{dt} dt \qquad (10)$$

The amount of fuel loaded on a rocket only determines the time for generating constant thrust, and this momentum impulse scheme completely avoids the energy conservation scheme. Therefore, the design speed of Dongfanghong-1 is relatively close to the measured speed in order to achieve a long lifespan of about 100 years.

We have reviewed multiple research papers on the efficiency of aerospace engines in recent years, and it seems that they are all avoiding the erroneous energy trap of the kinetic energy formula (1/2) $m\upsilon^2$.

4）Mass-energy relation

By the Encyclopædia Britannica, "If work, which transfers energy, is done on an object by applying a net force, the object speeds up and thereby gains kinetic energy. Translational kinetic energy of a body, (E_K) is equal to one-half the product of its mass m and the square of its velocity (v)." "The formula is valid only for low to relatively high speeds; for extremely high-speed particles it yields values that are too small."

The mass energy relationship equation $E=mc^2$ has been experimentally verified, and it is completely consistent in form with Leibniz's kinetic energy formula mv^2. When the speed gradually increases, even until it reaches the maximum limit of speed (the speed of light), the kinetic energy mv^2 can directly and accurately transition to mc^2.

Table 2. *Reference data for each sub level of Long March 1.*

	One stage	Second stage	Third stage
Stage length (m)	17.835	7.486	4.565
Diameter (m)	2.250	2.250	2.250
Stage mass (kg)	65250	13550	2200
Structural mass (kg)	4070	2270	400
Propellant mass (kg)	61070	11210	1800
Engine	YF-2	YF-3	FG-02
Propellant	Nitric acid-27S/		
dimethylhydrazine	Nitric acid-27S/		
dimethylhydrazine	Polysulfide rubber solid propellant		
Sea level thrust (kN)	1020	294.2	4.440
Sea level specific impulse (N·s/kg)	2349	2746	111.0
Working time (s)	140	102	2472

Conclusion

Compared to force, energy is a relatively abstract physical concept with numerous terms, the same dimensions, and a wide range of implications. The heat and work involved are process variables, with heat being related to thermal energy and work being related to mechanical energy. Generally speaking, heat and work are not state functions. When used as a state function, it is necessary to add constraints. If the constant pressure reaction heat is equal to the change in system enthalpy (H), ΔH; If work is used as a function to describe kinetic energy, it must refer to the kinetic energy at a velocity of $\upsilon/(\sqrt{2})$ in uniform variable speed motion. The kinetic energy is a state variable and a function of the state of moving object. The state function must be single valued, with each function value corresponding to a state. The work generated or consumed during free fall is (1/2) $m\upsilon^2=mgl$, which is a binary function corresponding to the velocities of two motion states: the maximum velocity υ and the average velocity $\upsilon/2$.

Helmholtz made significant contributions to physics, but in science, no amount of contribution can hide his mistakes. He proposed to call the work, (1/2) $m\upsilon^2$, the *vis viva,* and believed that this change was insignificant. He also used the height h as the displacement of the free fall, which led to the use of mgh to represent the potential energy, causing the potential energy of the starting point and the landing point to be reversed. His erroneous beliefs led to the erroneous expression of the law of conservation of mechanical energy later on. At the end of the 19th century, MacGregor, J. G., Greenhill, A. G., and C. G. K. advocated creating new words and forgetting old ones, replacing $m\upsilon^2$ with (1/2) $m\upsilon^2$. The three of them, along with the earlier Helmholtz, represented a trend in theoretical physics, which of course contained erroneous trends. Due to the abstraction of concepts such as the potential energy and the kinetic energy, their mistakes would not have had any significant consequences in their time. But in the 20th century and up to now, problems have arisen in both theory and practice regarding the derivation of the mass-energy relationship, the study of energy conservation in Kepler's planetary motion, the theoretical determination of thermal efficiency, the first and second cosmic velocities in the aerospace field, hypersonic vehicles, relativity, and quantum mechanics [11, 12, 13], all of which involve the conversion of other energy into kinetic energy and the calculation of velocity by the kinetic energy. Today we discovered a problem and cannot let it continue to affect our next generation. We hope to include as few errors as possible in the basic scientific theories of our time.

In summary, the manipulation of Leibniz's *vis viva* formula by Helmholtz and others, as well as the erroneous expression of the law of conservation of mechanical energy, have had a profound impact in theoretical physics and must be corrected. The (1/2) $m\upsilon^2$ is not kinetic energy, but only one-half of the kinetic energy.

Acknowledgements

The authors would like to thank Teacher Huang from Changtian Middle School in Zhaoqing for lending us the current high school physics textbook，and professor Kui Fu Chen in Applied Mechanics Department, China Agricultural University for discussion of the kinetic energy.

Author contributions

Xiao En Wang, Deng guo Wang and Yin Bo Wang wrote the paper.

Competing interests

The authors declares no competing interest.

Data Availability Statement

There is no availability data other than this manuscript.

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